The Co-ordination Principles: a Problem for Bilateralism

[Discussion note]

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The division of thoughts (judgements) into
affirmative and negative is of no use to logic,
and I doubt if it can be carried through.
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Notes for Ludwig Darmstaedter

Ian Rumfitt (2000) has recently proposed a new conception of the sense of the logical words according to which classical logic can be justified. (See also the further elaboration in Rumfitt 2002 in a reply to criticism.) The conception follows the general idea that the sense of a sentence is determined by its use in language. For molecular declarative sentences, built from sub-sentences by means of a propositional logical operator, the sense of the complex sentence is determined by the senses of the ingredient sentences and the contribution made by the logical operator. It has been plausibly argued that the latter is specified by introduction and elimination rules. The traditional discussion of these matters labours under the assumption that the sense of a sentence is determined by the conditions under which it can be correctly *asserted*. Michael Dummett and others have argued that, under this conception, *only* intuitionistic logic can be justified in general. Classical logic is not justified because the classical elimination and introduction rules for the negation operator are not in *harmony*.[1]

The conception of Rumfitt follows a *use based* account of sense and does respect the requirements of harmony (and stability) between the introduction and elimination rules of the logical words. Harmony (and stability) is achieved due to a novel conception of sense, viz. that the use of sentences must take into consideration not only the conditions under which they may correctly be asserted but also under which they may be correctly *denied*. In Rumfitt's felicitous terminology, it is adopted a *bilateral* conception of sense, contra the traditional *unilateral* conception. The thesis is that bilateralism is able to give a use based justification of classical logic. One must not mistake Rumfitt's arguments for an argument for classical logic in *all* areas of discourse. Rather, it is an argument *against* the view, championed by Dummett and others, that intuitionistic logic is *au fond* the logic of every area of discourse (even when the discourse happens to conform to the law of excluded middle – a feature which may arise in a particular area of discourse because, within that area, it is always the case that a sentence, or its negation, can be correctly asserted). I sympathize with Rumfitt's conclusion against the Dummettian view, but question his way out. Contrary

[1]For an explanation of this and related matters (e.g. the requirement of stability) see Dummett 1991, specially p. 291.

to Dummett (2002), who doubts the coherence of the bilateral approach to start with (being skeptical about the possibility of a non-question begging account of the classical understanding of negation), I grant its intelligibility only to raise a technical problem – possibly a serious difficulty for bilateralism.

In a bilateral theory of sense, the force of assertion and denial are primitive, symmetric features of speech acts. In particular, the denial of a sentence is not to be explained – as in unilateralist theories – as the assertion of the negation of the sentence. It is the other way around for the bilateralist: the sense of negation is determined by rules of introduction and elimination which rely upon the illocutionary forces of both assertion and denial.[2] We follow Rumfitt's terminology and use *signed sentences* $+A$ and $-A$ as formal correlates of the operation of forming an interrogative from a declarative sentence A and appending the answer 'Yes', respectively 'No'.[3] The signalling cannot be iterated, e.g. $+(-A)$ or $-(-A)$ are not well-formed sentences (by the way, this feature syntactically distinguishes the '$-$' sign from the negation sign '$\neg$'). Where α is a signed sentence, let α^* be the signed sentence obtained by reversing the sign of α. Note that $(\alpha^*)^*$ is the *same* sentence as α. We use a notation of the form $\Gamma \vdash \alpha$ to say that the signed sentence α has a natural deduction proof from (open) assumptions lying within the (finite) set of signed sentences Γ. We also allow α to be the punctuation sign '$\perp$' (the insertion of this sign is just a colourful way to mark that a

[2] We assume familiarity with theses rules, as well as with the rules of the remaining propositional connectives, as specified in Rumfitt 2000 on pp. 800-2.

[3] For the sake of convenience, we formulate the introduction and elimination rules of negation: [$+\neg$I] from $-A$ infer $+(\neg A)$; [$+\neg$E] from $+(\neg A)$ infer $-A$; [$-\neg$I] from $+A$ infer $-(\neg A)$; and [$-\neg$E] from $-(\neg A)$ infer $+A$.

certain proof configuration has been reached.) As usual, Rumfitt's calculus has introduction and elimination rules for the propositional connectives. In addition, it also has *co-ordination principles* that govern the relationship between correctly asserting and correctly denying a sentence. These principles are necessary for endowing sentences with a *coherent* bilateral sense because, for instance, the conditions for correctly asserting a sentence must not intersect with the conditions for correctly denying it. Notice that there are no co-ordination principles in unilateral theories of sense: in unilateralism there is nothing to co-ordinate. Rumfitt says that 'plainly, inference rules for connectives cannot ensure that the atomic sentences of the relevant language meet this condition [viz. that the conditions for correctly asserting a sentence do not intersect with the conditions for correctly denying it]. They can, however, ensure that if the language's atomic sentences meet it then molecular sentences will meet it too. They can, one might put it, ensure that *co-ordination is preserved*' (Rumfitt's italics). (Rumfitt 2002, p. 308.) Rumfitt proposes the following two co-ordination principles:

(C1) if $\Gamma \vdash \alpha$ and $\Gamma \vdash \alpha^*$ then $\Gamma \vdash \bot$;

(C2) if $\Gamma, \beta \vdash \bot$ then $\Gamma \vdash \beta^*$.

(Rumfitt 2000, p. 804.) The first co-ordination principle indicates that accepting α and α^* marks a dead end, a dead end from which one can escape by discharging one assumption according to the second co-ordination principle. On the intended bilateralist reading of the formalism, the second co-ordination principle mirrors (at the atomic level) a symmetry between asserting and denying a sentence – and this is the hallmark of bilateralism.

For the sake of the argument, we accept both co-ordination principles for signed atomic sentences (i.e. for α and β of the form $+P$ or $-P$, with P an atomic sentence). In other words, we presuppose that the bilateralist has an account for correctly asserting and for correctly denying atomic sentences (in a given area of discourse) such that (C1) and (C2) hold for signed atomic sentences α.[4] Otherwise, the account for correctly asserting and correctly denying *molecular* sentences is fully specified *from* the atomic case *by means of* the introduction and elimination rules of the logical operators.

In Rumfitt's calculus, the law of double negation elimination is easily derivable. Nevertheless, this is not enough to sustain classical logic. Quite to the contrary, we want to draw attention to the fact that Rumfitt's logical system is seriously *paralysed* if the co-ordination principles do not hold for *all* sentences of the language. It is easy to show that $\vdash +(A \vee \neg A)$ is itself a co-ordination principle of type (C2), namely the co-ordination principle that permits to move from the sequent $-(A \vee \neg A) \vdash \bot$ to the sequent $\vdash +(A \vee \neg A)$. This is so because the sequent $-(A \vee \neg A) \vdash \bot$ can be derived using only the introduction and elimination rules. Dually, $\vdash -(A \& \neg A)$ is also a co-ordination principle of type (C2), the one that permits to obtain the sequent $\vdash -(A \& \neg A)$ from the sequent $+(A \& \neg A) \vdash \bot$. Therefore, both the law of excluded middle and the law of non-contradiction are co-ordination principles of type (C2). Surprisingly, whereas the first co-ordination principle (C1) receives careful attention in Rumfitt's article (and is preserved by the propositional connectives), the second co-ordination principle (C2) is

[4] Actually, (C1) is superfluous for the *atomic* level under Rumfitt's understanding of '$\bot$'.

hardly discussed. In the sequel, I show that the co-ordination principle (C2) is not preserved. Indeed, I will show that Rumfitt's introduction and elimination rules together with the co-ordination principles for signed atomic sentences do not entail $-(A\&\neg A)$, even for atomic A.

1 The counterexample

In this section we describe a Kripke model counterexample to the preservation of the co-ordination principle (C2). A Kripke structure $\mathcal{W}$ for Rumfitt's signed calculus consists of the following data: a) a non-empty set W of possible worlds; b) an accessibility relation $\leq$ in W, i.e. a reflexive and transitive binary relation in W; c) valuation maps v^+ and v^- that assign to each propositional letter P a subset of W. Intuitively, $v^+(P)$ is the set of worlds in which P is true, and $v^-(P)$ is the set of worlds in which P is false. Moreover, the following three clauses must hold for all propositional letters P:

(i) $v^+(P) \cap v^-(P) = \emptyset$;

(ii) $\forall w \forall w'$(if $w \in v^+(P)$ and $w \leq w'$ then $w' \in v^+(P)$);

(iii) $\forall w \forall w'$(if $w \in v^-(P)$ and $w \leq w'$ then $w' \in v^-(P)$).

Definition. *Given a Kripke structure $\mathcal{W}$, w a world, and A a propositional sentence, we simultaneously define $w \models^+_{\mathcal{W}} A$ and $w \models^-_{\mathcal{W}} A$ according to the following recursive clauses:*

1. $w \models^+_{\mathcal{W}} P$ *iff* $w \in v^+(P)$, *for propositional letters P;*

2. *$w \models^-_{\mathcal{W}} P$ iff $w \in v^-(P)$, for propositional letters P;*

3. *$w \models^+_{\mathcal{W}} A \vee B$ iff $w \models^+_{\mathcal{W}} A$ or $w \models^+_{\mathcal{W}} B$;*

4. *$w \models^-_{\mathcal{W}} A \vee B$ iff $w \models^-_{\mathcal{W}} A$ and $w \models^-_{\mathcal{W}} B$;*

5. *$w \models^+_{\mathcal{W}} A \& B$ iff $w \models^+_{\mathcal{W}} A$ and $w \models^+_{\mathcal{W}} B$;*

6. *$w \models^-_{\mathcal{W}} A \& B$ iff $w \models^-_{\mathcal{W}} A$ or $w \models^-_{\mathcal{W}} B$;*

7. *$w \models^+_{\mathcal{W}} A \to B$ iff $\forall w'$(if $w \leq w'$ and $w' \models^+_{\mathcal{W}} A$ then $w' \models^+_{\mathcal{W}} B$);*

8. *$w \models^-_{\mathcal{W}} A \to B$ iff $w \models^+_{\mathcal{W}} A$ and $w \models^-_{\mathcal{W}} B$;*

9. *$w \models^+_{\mathcal{W}} \neg A$ iff $w \models^-_{\mathcal{W}} A$; and*

10. *$w \models^-_{\mathcal{W}} \neg A$ iff $w \models^+_{\mathcal{W}} A$.*[5]

We now say that $w \models_{\mathcal{W}} +A$ if $w \models^+_{\mathcal{W}} A$, and $w \models_{\mathcal{W}} -A$ if $w \models^-_{\mathcal{W}} A$. In this way, the relation $w \models_{\mathcal{W}} \alpha$ is defined for every world w and every signed sentence α. The following result is easy to prove:

Lemma. *Let $\mathcal{W}$ be a Kripke structure. If w, w' are worlds with $w \leq w'$, α is a signed sentence, and $w \models_{\mathcal{W}} \alpha$, then $w' \models_{\mathcal{W}} \alpha$.*

Given a finite set Γ of signed sentences $\{\alpha_1, \ldots, \alpha_k\}$ and a world w, we let $w \models_{\mathcal{W}} \Gamma$ abbreviate $w \models_{\mathcal{W}} \alpha_1$, $w \models_{\mathcal{W}} \alpha_2$, $\ldots$, $w \models_{\mathcal{W}} \alpha_k$.

Definition. *Let Γ be a finite set of signed sentences and α a signed sentence. Let $\mathcal{W}$ be a Kripke structure. We say that $\Gamma \models_{\mathcal{W}} \alpha$ if the following holds: $\forall w \in W$ (if $w \models_{\mathcal{W}} \Gamma$ then $w \models_{\mathcal{W}} \alpha$).*

[5]This is the Kripke semantics defined by Seiki Akama (1986). Note that Akama uses the negation sign '$\sim$' where we use '$\neg$'. He reserves the latter sign for another notion.

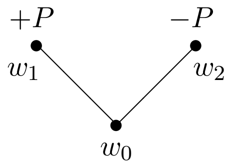
In the sequel, we write $\Gamma \vdash^* \alpha$ for saying that there is a natural deduction proof of α from Γ in Rumfitt's calculus *without* the co-ordination principles. The following soundness theorem is instrumental:

Theorem. *Let Γ be a finite set of signed sentences and α a signed sentence. If $\Gamma \vdash^* \alpha$ then, for every Kripke structure $\mathcal{W}$, $\Gamma \models_{\mathcal{W}} \alpha$.*

The proof is, as usual, by induction on the length of the derivations. One needs to go through all the introduction and elimination rules of Rumfitt's calculus, and check that the corresponding semantical property holds. We check the $[-\neg\text{E}]$ rule, viz. if $\Gamma \vdash^* -\neg A$ then $\Gamma \vdash^* +A$. Let $\mathcal{W}$ be a Kripke structure and suppose that $\Gamma \models_{\mathcal{W}} -\neg A$. Take a world w such that $w \models_{\mathcal{W}} \Gamma$. According to the supposition, $w \models_{\mathcal{W}} -\neg A$ and, therefore, $w \models^{-}_{\mathcal{W}} \neg A$. Using clause 10 of the semantical definition above, we get $w \models^{+}_{\mathcal{W}} A$, i.e. $w \models_{\mathcal{W}} +A$. By the arbitrariness of w, we have showed that $\Gamma \models_{\mathcal{W}} +A$.

We follow Rumfitt's idiosyncrasy according to which the symbol '$\perp$' is merely a punctuation sign indicating that one has reached deductions of α and α^*, for some signed atomic sentence α. On this reading, the co-ordination principle (C1) is superfluous for atomic sentences (and it can be proved for formulas in general). Let us fix a Kripke structure $\mathcal{W}$. Because of condition (i), $\Gamma \models_{\mathcal{W}} \perp$ says that there is no world w such that $w \models_{\mathcal{W}} \Gamma$. Therefore, in semantical terms, the co-ordination principle (C2) says that if there is no world w such that $w \models_{\mathcal{W}} \Gamma \cup \{\beta\}$ then $\Gamma \models_{\mathcal{W}} \beta^*$.

We are now ready to give the counterexample $\mathcal{U}$. It is:



In the above, P is a propositional letter (we ignore the others). The Kripke structure $\mathcal{U}$ has three worlds $\{w_0, w_1, w_2\}$, ordered according to the picture above ($w_0 \leq w_1$ and $w_0 \leq w_2$), and whose valuations are $v^+(P) = \{w_1\}$ and $v^-(P) = \{w_2\}$.

We claim that this structure satisfies the co-ordination principle (C2) for $\beta = +P$ and $\beta = -P$. Let Γ be a (finite) set of sentences and suppose that there is no world $w \in \{w_0, w_1, w_2\}$ such that $w \models_{\mathcal{U}} \Gamma \cup \{+P\}$. Let $u \models_{\mathcal{U}} \Gamma$. By the lemma, for u' with $u \leq u'$ one also has $u' \models_{\mathcal{U}} \Gamma$ and, therefore, $u' \not\models_{\mathcal{U}} +P$. In other words, there is no world accessible from u in which $+P$ holds. This entails that u must be w_2. Since $v^-(P) = \{w_2\}$, we get $u \models_{\mathcal{U}} -P$. By the arbitrariness of u, we proved that $\Gamma \models_{\mathcal{U}} -P$. A symmetrical argument also holds for $-P$ instead of $+P$.

We have showed that $\mathcal{U}$ satisfies the co-ordination principles at the atomic level. However, it does not satisfy them in general. This can be seen by checking that $w_0 \not\models_{\mathcal{U}} -(P\&\neg P)$.

2 Coda

Rumfitt (2000) proposes a new conception of the sense of the logical words according to which classical logic is justified. This conception is based on the novel idea that the uses of sentences must take into account not only the conditions under which they may correctly be asserted but also under which they may be correctly denied. This binary feature is the main characteristic of the bilateral approach to sense. A necessary condition for a sentence to have a coherent bilateral sense is that the acts of asserting it and rejecting it should be co-ordinated. Inference rules for connectives cannot ensure that the atomic sentences are co-ordinated. This is something that must be formally postulated (and argued for *in concreto*). Since the sense of a molecular sentence must be *fully* determined by the introduction and elimination rules of its principal connective (given the conditions for asserting and denying the ingredient sentences), the co-ordination principles for arbitrary sentences must not be postulated. They should rather follow from the rules and the co-ordination at the atomic level. This is not the case, however. As a consequence, some very basic laws of logic are unaccounted for, posing a serious problem for Rumfitt's bilateralism.

References

[1] S. Akama 1986: 'Constructive Predicate Logic with Strong Negation and Model Theory'. *Notre Dame Journal of Formal Logic*, 29, pp. 18-27.

[2] M. Dummett 1991: *The Logical Basis of Metaphysics*. Harvard University Press, Cambridge, Massachusetts, 1991.

[3] M. Dummett 2002: "Yes', 'No' and 'Can't Say". *Mind*, 111, pp. 289-295.

[4] I. Rumfitt 2000: "Yes' and 'No". *Mind*, 109, pp. 781-823.

[5] I. Rumfitt 2002: 'Unilateralism Disarmed: A Reply to Dummett and Gibbard'. *Mind*, 111, pp. 305-321.