CHAPTER 1 PROBLEMS

1.1 Determine whether the element in Fig. 1.1 is absorbing or supplying power and how much.

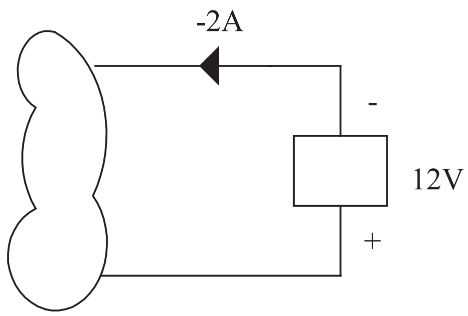

Fig. 1.1

1.2 In Fig. 1.2, element 2 absorbs 24W of power. Is element 1 absorbing or supplying power and how much.

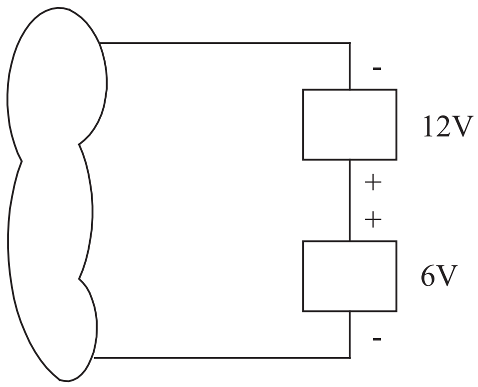

Fig. 1.2

1.3. Given the network in Fig.1.3 find the value of the unknown voltage V_X.

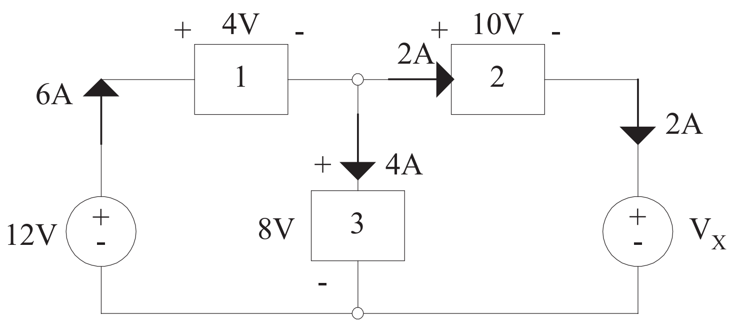

Fig. 1.3

CHAPTER 1 SOLUTIONS

1.1 One of the easiest ways to examine this problem is to compare it with the diagram that illustrates the sign convention for power as shown below in Fig. S1.1(b).

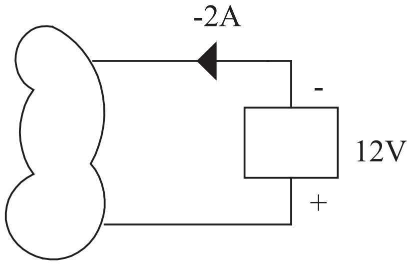

Fig. S1.1(a)

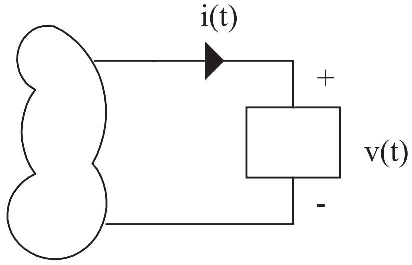

Fig. S1.1(b)

We know that if we simply arrange our variables in the problem to match those in the diagram on the right, then p(t) = i(t) v(t) and the resultant sign will indicate if the element is absorbing (+ sign) or supplying (- sign) power.

If we reverse the direction of the current, we must change the sign and if we reverse the direction of the voltage we must change the sign also. Therefore, if we make the diagram in Fig. S1.1(a) to look like that in Fig. S1.1(b), the resulting diagram is shown in Fig. S1.1(c).

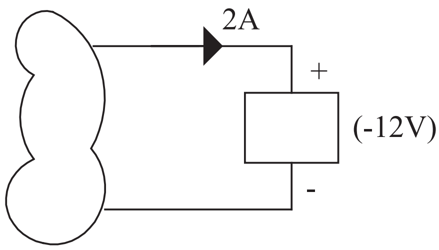

Fig. S1.1(c)

Now the power is calculated as

$$P = (2)\ (-12) = -24W$$

And the negative sign indicates that the element is supplying power.

1.2 Recall that the diagram for the passive sign convention for power is shown in Fig. S1.2(a) and if p = vi is positive the element is absorbing power and if p is negative, power is being supplied by the element.

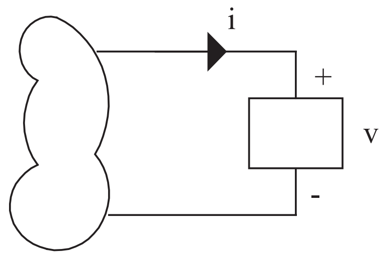

Fig. S1.2(a)

If we now isolate the element 2 and examine it, since it is absorbing power, the current must enter the positive terminal of this element. Then

$$P = VI$$
$$24 = 6(I)$$
$$I = 4A$$

The current entering the positive terminal of element 2 is the same as that leaving the positive terminal of element 1. If we now isolate our discussion on element 1, we find that the voltage across the element is 6V and the current of 4A emanates from the positive terminal. If we reverse the current, and change its sign, so that the isolated element looks like the one in Fig. S1.2(a), then

$$P = (6)\ (-4) = -24W$$

And element 1 is supplying 24W of power.

1.3 By employing the sign convention for power, we can determine whether each element in the diagram is absorbing or supplying power. Then we can apply the principle of the conservation of energy which means that the power supplied must be equal to the power absorbed.

If we now isolate each element and compare it to that shown in Fig. S1.3(a) for the sign convention for power, we can determine if the elements are absorbing or supplying power.

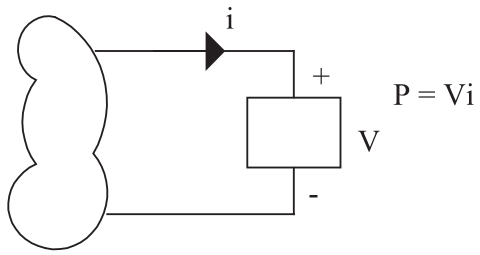

Fig. S1.3(a)

For the 12V source and the current through it to be arranged as shown in Fig. S1.3(a), the current must be reversed and its sign changed. Therefore

$$P_{12V} = (12)\ (-6) = -72W$$

Treating the remaining elements in a similar manner yields

$$P_1 = (4)\,(6) = 24W$$
$$P_2 = (2)\,(10) = 20W$$
$$P_3 = (8)\,(4) = 32W$$
$$P_{VX} = (V_X)\,(2) = 2V_X$$

Applying the principle of the conservation of energy, we obtain

$$-72 + 24 + 20 + 32 + 2V_X = 0$$

And

$$V_X = -2V$$

CHAPTER 2 PROBLEMS

2.1 Determine the voltages V_1 and V_2 in the network in Fig. 2.1 using voltage division.

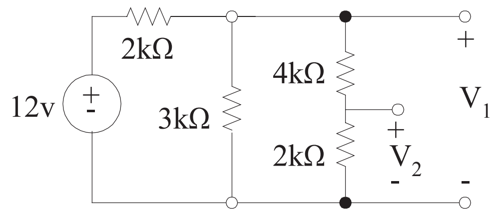

Fig. 2.1

2.2 Find the currents I_1 and I_0 in the circuit in Fig. 2.2 using current division.

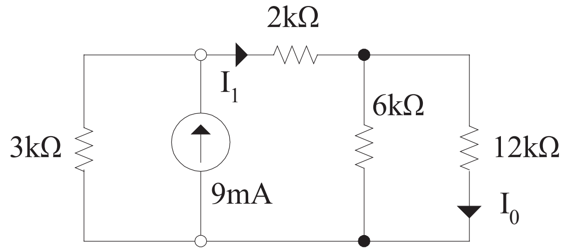

Fig. 2.2

2.3 Find the resistance of the network in Fig. 2.3 at the terminals A-B.

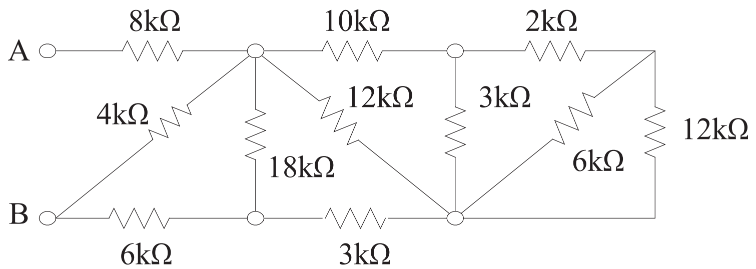

Fig. 2.3

2.4 Find the resistance of the network shown in Fig. 2.4 at the terminals A-B.

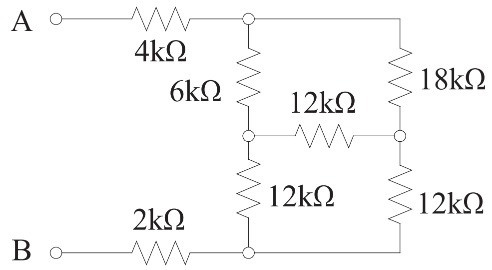

Fig. 2.4

2.5 Find all the currents and voltages in the network in Fig. 2.5.

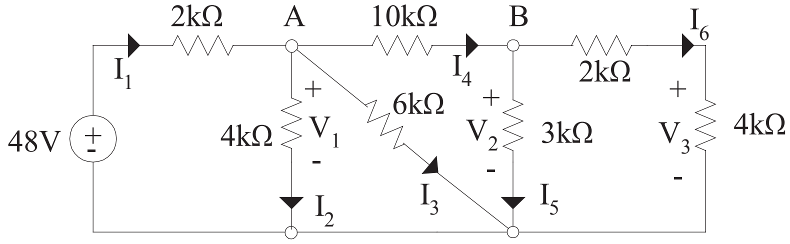

Fig. 2.5

2.6 In the network in Fig. 2.6, the current in the 4kΩ resistor is 3mA. Find the input voltage V_S.

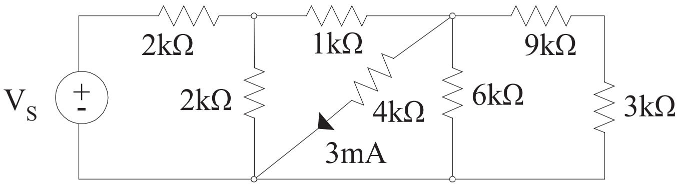

Fig. 2.6

CHAPTER 2 SOLUTIONS

2.1 We recall that if the circuit is of the form

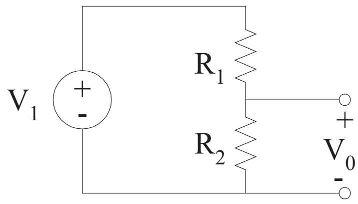

Fig. S2.1(a)

Then using voltage division

$$V_0 = \left(\frac{R_2}{R_1 + R_2} \right) V_1$$

That is the voltage V_1 divides between the two resistors in direct proportion to their resistances. With this in mind, we can draw the original network in the form

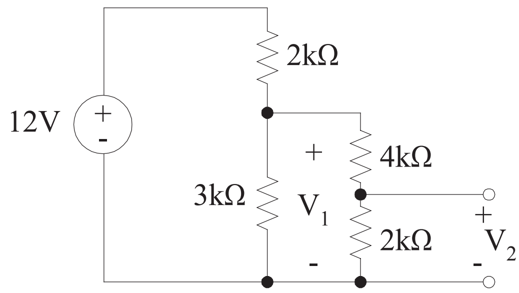

Fig. S2.1(b)

The series combination of the 4kΩ and 2kΩ resistors and their parallel combination with the 3kΩ resistor yields the network in Fig. S2.1(c).

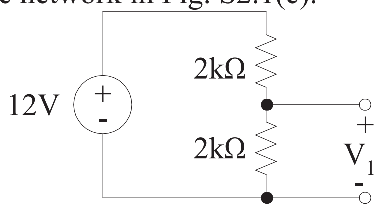

Fig. S2.1(c)

Now voltage division can be sequentially applied. From Fig. S2.1(c).

$$V_1 = \left(\frac{2k}{2k + 2k} \right) 12$$
$$= 6V$$

Then from the network in Fig. S2.1(b)

$$V_2 = \left(\frac{2k}{2k + 4k}\right)V_1$$
$$= 2V$$

2.2 If we combine the 6k and 12k ohm resistors, the network is reduced to that shown in Fig. S2.2(a).

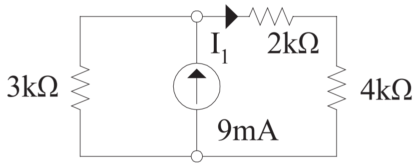

Fig. S2.2(a)

The current emanating from the source will split between the two parallel paths, one of which is the 3kΩ resistor and the other is the series combination of the 2k and 4kΩ resistors. Applying current division

$$I_1 = \frac{9}{k}\left(\frac{3k}{3k + (2k + 4k)}\right)$$
$$= 3mA$$

Using KCL or current division we can also show that the current in the 3kΩ resistor is 6mA. The original circuit in Fig. S2.2 (b) indicates that I_1 will now be split between the two parallel paths defined by the 6k and 12k-Ω resistors.

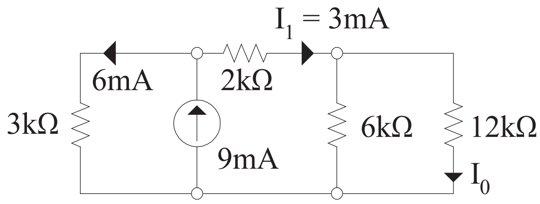

Fig. S2.2(b)

Applying current division again

$$I_0 = I_1\left(\frac{6k}{6k + 12k}\right)$$
$$I_0 = \frac{3}{k}\left(\frac{6k}{18k}\right)$$
$$= 1mA$$

Likewise the current in the 6kΩ resistor can be found by KCL or current division to be 2mA. Note that KCL is satisfied at every node.

2.3 To provide some reference points, the circuit is labeled as shown in Fig. S2.3(a).

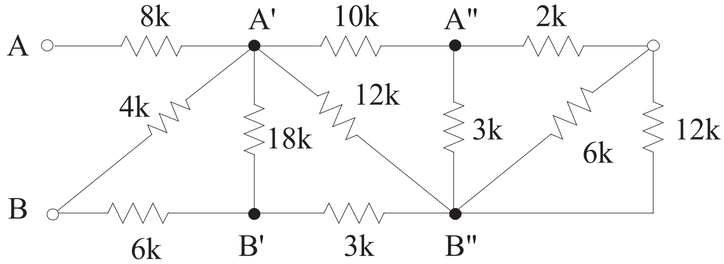

Fig. S2.3(a)

Starting at the opposite end of the network from the terminals A-B, we begin looking for resistors that can be combined, e.g. resistors that are in series or parallel. Note that none of the resistors in the middle of the network can be combined in anyway. However, at the right-hand edge of the network, we see that the 6k and 12k ohm resistors are in parallel and their combination is in series with the 2kΩ resistor. This combination of 6k || 12k + 2k is in parallel with the 3kΩ resistor reducing the network to that shown in Fig. S2.3(b).

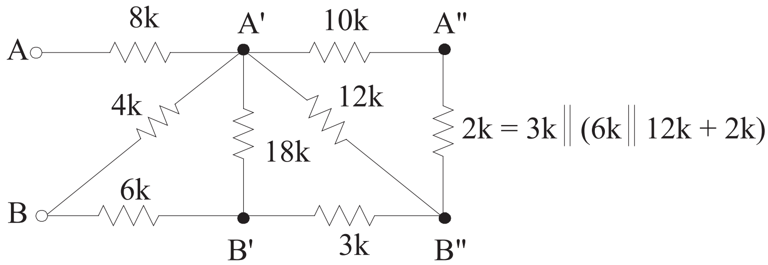

Fig. S2.3(b)

Repeating this process, we see that the 2kΩ resistor is in series with the 10kΩ resistor and that combination is in parallel with the12kΩ resistor. This equivalent 6kΩ resistor (2k + 10k) || 12k is in series with the 3kΩ resistor and that combination is in parallel with the 18kΩ resistor that (6k + 3k) || 18k = 6kΩ and thus the network is reduced to that shown in Fig. S2.3(c).

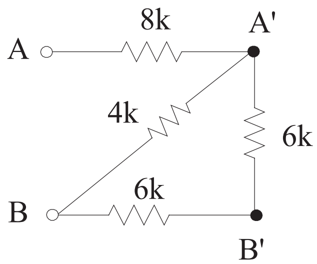

Fig. S2.3(c)

At this point we see that the two 6kΩ resistors are in series and their combination in parallel with the 4kΩ resistor. This combination (6k + 6k)|| 4k = 3kΩ which is in series with 8kΩ resistors yielding A total resistance R_{AB} = 3k + 8k = 11kΩ.

2.4 An examination of the network indicates that there are no series or parallel combinations of resistors in this network. However, if we redraw the network in the form shown in Fig. S2.4(a), we find that the networks have two deltas back to back.

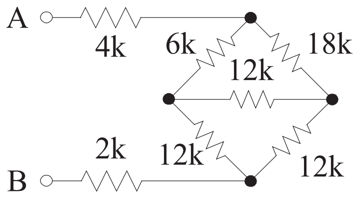

Fig. S2.4(a)

If we apply the Δ→Y transformation to either delta, the network can be reduced to a circuit in which the various resistors are either in series or parallel. Employing the Δ→Y transformation to the upper delta, we find the new elements using the following equations as illustrated in Fig. S2.4(b)

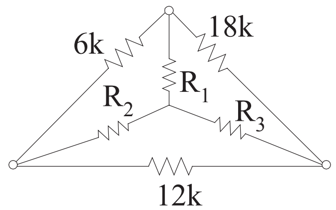

Fig. S2.4(b)

$$R_1 = \frac{(6k)(18k)}{6k + 12k + 18k} = 3k\Omega$$

$$R_2 = \frac{(6k)(12k)}{6k + 12k + 18k} = 2k\Omega$$

$$R_3 = \frac{(12k)(18k)}{6k + 12k + 18k} = 6k\Omega$$

The network is now reduced to that shown in Fig. S2.4(c).

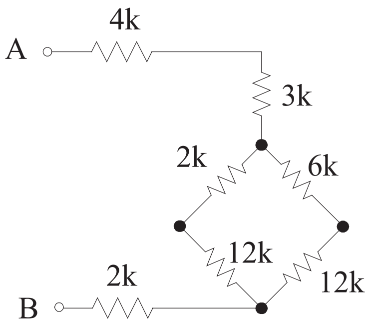

Fig. S2.4(c)

Now the total resistance, R_{AB} is equal to the parallel combination of (2k + 12k) and (6k + 12k) in series with the remaining resistors i.e.

$$R_{AB} = 4k + 3k + (14k || 18k) + 2k$$
$$= 16.875k\Omega$$

If we had applied the $\Delta \rightarrow Y$ transformation to the lower delta, we would obtain the network in Fig. S2.4(d).

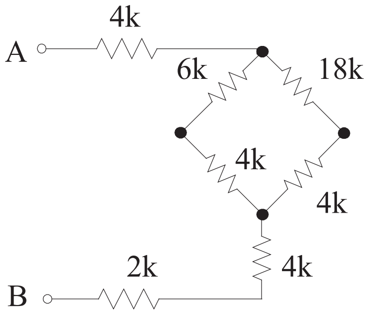

Fig. S2.4(d)

In this case, the total resistance R_{AB} is

$$R_{AB} = 4k + (6k + 4k) || (18k + 4k) + 4k + 2k$$
$$= 16.875k\Omega$$

which is, of course, the same as our earlier result.

2.5 Our approach to this problem will be to first find the total resistance seen by the source, use it to find I_1 and then apply Ohm's law, KCL, KVL, current division and voltage division to determine the remaining unknown quantities. Starting at the opposite end of the network from the source, the 2k and 4k ohm resistors are in series and that combination is in parallel with the 3kΩ resistor yielding the network in Fig. S2.5(a).

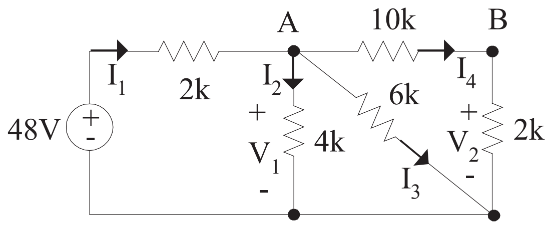

Fig. S2.5(a)

Proceeding, the 2k and 10k ohm resistors are in series and their combination is in parallel with both the 4k and 6k ohm resistors. The combination (10k + 2k)|| 6k|| 4k = 2kΩ. Therefore, this further reduction of the network is as shown in Fig. S2.5(b).

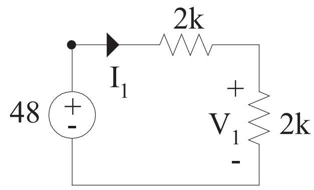

Fig. S2.5(b)

Now I_1 and V_1 can be easily obtained.

$$I_1 = \frac{48}{2k + 2k} = 12mA$$

And by Ohm's law

$$\begin{aligned} V_1 &= 2kI_1 \\ &= 24V \end{aligned}$$

or using voltage division

$$\begin{aligned} V_1 &= 48\left(\frac{2k}{2k + 2k}\right) \\ &= 24V \end{aligned}$$

once V_1 is known, I_2 and I_3 can be obtained using Ohm's law

$$I_2 = \frac{V_1}{4k} = \frac{24}{4k} = 6mA$$

$$I_3 = \frac{V_1}{6k} = \frac{24}{6k} = 4mA$$

I_4 can be obtained using KCL at node A. As shown on the circuit diagram.

$$I_1 = I_2 + I_3 + I_4$$

$$\frac{12}{k} = \frac{6}{k} + \frac{4}{k} + I_4$$

$$I_4 = \frac{2}{k} = 2mA$$

The voltage V_2 is then

$$\begin{aligned} V_2 &= V_1 - 10kI_4 \\ &= 24 - (10k)\left(\frac{2}{k}\right) \\ &= 4V \end{aligned}$$

or using voltage division

$$\begin{aligned} V_2 &= V_1\left(\frac{2k}{10k + 2k}\right) \\ &= 24\left(\frac{1}{6}\right) \\ &= 4V \end{aligned}$$

Knowing V_2, I_5 can be derived using Ohm's law

$$\begin{aligned} I_5 &= \frac{V_2}{3k} \\ &= \frac{4}{3}mA \end{aligned}$$

and also

$$\begin{aligned} I_6 &= \frac{V_2}{2k + 4k} \\ &= \frac{2}{3}mA \end{aligned}$$

current division can also be used to find I_5 and I_6.

$$\begin{aligned} I_5 &= I_4\left(\frac{2k + 4k}{2k + 4k + 3k}\right) \\ &= \frac{4}{3}mA \end{aligned}$$

and

$$I_6 = I_4\left(\frac{3k}{3k + 2k + 4k}\right)$$
$$= \frac{2}{3}mA$$

Finally V_3 can be obtained using KVL or voltage division

$$V_3 = V_2 - 2kI_6$$
$$= 4 - 2k\left(\frac{2}{3k}\right)$$
$$= \frac{8}{3}V$$

and

$$V_3 = V_2\left(\frac{4k}{4k + 2k}\right)$$
$$= \frac{8}{3}V$$

2.6 The network is labeled with all currents and voltages in Fig. S2.6.

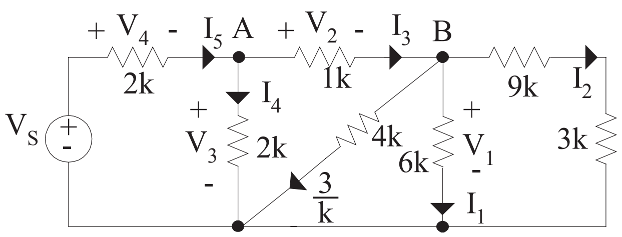

Fig. S2.6

Given the 3mA current in the 4kΩ resistor, the voltage

$$V_1 = \left(\frac{3}{k}\right)(4k) = 12V$$

Now knowing V_1, I_1 and I_2 can be obtained using Ohm's law as

$$I_1 = \frac{V_1}{6k} = \frac{12}{6k} = 2mA$$
$$I_2 = \frac{V_1}{9k + 3k} = \frac{12}{12k} = 1mA$$

Applying KCL at node B

$$I_3 = \frac{3}{k} + I_1 + I_2$$
$$= 6\text{mA}$$

Then using Ohm’s law

$$V_2 = I_3\,(1\text{k})$$
$$= 6\text{V}$$

KVL can then be used to obtain V_3 i.e.

$$V_3 = V_2 + V_1$$
$$= 6 + 12$$
$$= 18\text{V}$$

Then

$$I_4 = \frac{V_3}{2\text{k}}$$
$$= 9\text{mA}$$

And

$$I_5 = I_3 + I_4$$
$$= \frac{6}{k} + \frac{9}{k}$$
$$= 15\text{mA}$$

using Ohm’s law

$$V_4 = (2\text{k})\,I_5$$
$$= 30\text{V}$$

and finally

$$V_S = V_4 + V_3$$
$$= 48\text{V}$$

CHAPTER 4 PROBLEMS

4.1 Derive the gain equation for the nonideal noninverting op-amp configuration and show that it reduces to the ideal gain equation if R_i and A are very large, i.e. greater than 10^6.

4.2 Determine the voltage gain of the op-amp circuit shown in Fig. 4.2.

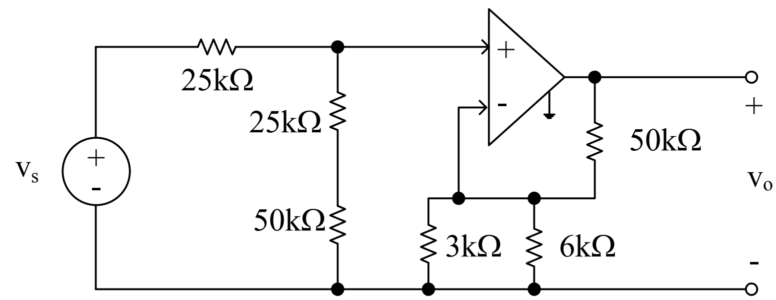

Fig. 4.2

4.3 Using the ideal op-amp model show that for the circuit shown in Fig. 4.3, the output voltage is directly related to any small change ΔR.

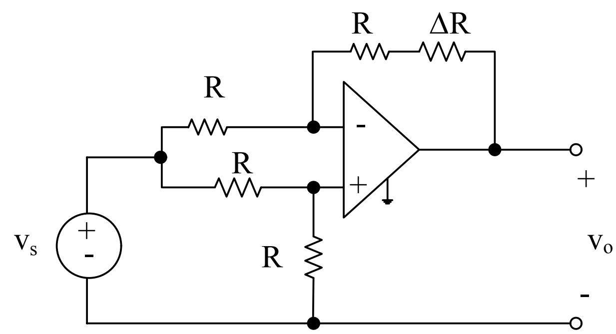

Fig. 4.3

4.4 Given an op-amp and seven standard 12kΩ resistors, design an op-amp circuit that will produce an output of

$$v_0 = -2v_1 - \frac{1}{2}v_2$$

CHAPTER 4 SOLUTIONS

4.1 The noninverting op-amp circuit is shown in Fig. S4.1(a).

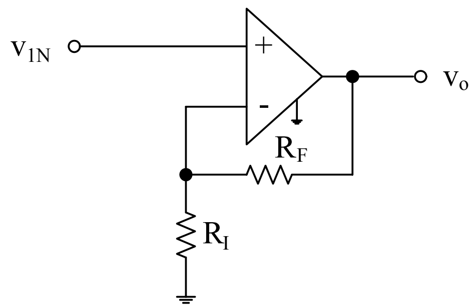

Fig. S4.1(a)

The nonideal model is

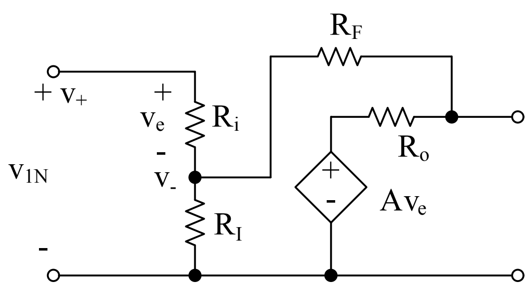

Fig. S4.1(b)

or

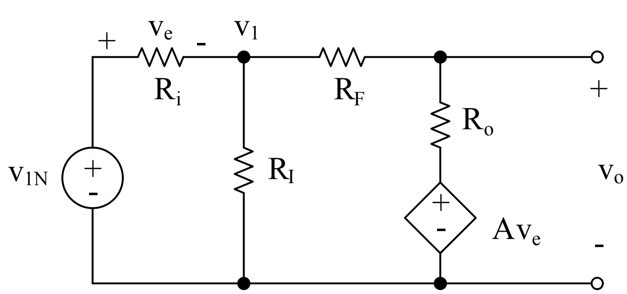

Fig. S4.1(c)

The node equations for this circuit are

$$\frac{v_1 - v_{IN}}{R_i} + \frac{v_1}{R_I} + \frac{v_1 - v_o}{R_F} = 0$$

$$\frac{v_o - v_1}{R_F} + \frac{v_o - Av_e}{R_o} = 0$$

$$v_e = v_{1N} - v_1$$

or

$$\left[\frac{1}{R_i} + \frac{1}{R_I} + \frac{1}{R_F}\right] v_1 - \left[\frac{1}{R_F}\right] v_o = \frac{v_{1N}}{R_i}$$

$$-\left[\frac{1}{R_F} - \frac{A}{R_o}\right] v_1 + \left[\frac{1}{R_F} + \frac{1}{R_o}\right] v_o = \frac{Av_{1N}}{R_o}$$

Following the development on page 141 of the text yields

$$v_o = \frac{\left[\frac{1}{R_F} - \frac{A}{R_o}\right]\frac{v_{1N}}{R_i} + \left[\frac{1}{R_i} + \frac{1}{R_I} + \frac{1}{R_F}\right]\frac{Av_{1N}}{R_o}}{\left(\frac{1}{R_i} + \frac{1}{R_I} + \frac{1}{R_F}\right)\left(\frac{1}{R_F} + \frac{1}{R_o}\right) - \frac{1}{R_F}\left(\frac{1}{R_F} - \frac{A}{R_o}\right)}$$

assuming $R_i \to \infty$, the equation reduces to

$$\frac{v_o}{v_{1N}} = \frac{\left(\frac{1}{R_I} + \frac{1}{R_F}\right)\left(\frac{A}{R_o}\right)}{\left(\frac{1}{R_I} + \frac{1}{R_F}\right)\left(\frac{1}{R_F} + \frac{1}{R_o}\right) - \frac{1}{R_F}\left(\frac{1}{R_F} - \frac{A}{R_o}\right)}$$

Now dividing both numerator and denominator by A and using $A \to \infty$ yields

$$\frac{v_o}{v_{1N}} = \frac{\frac{1}{R_o}\left(\frac{1}{R_I} + \frac{1}{R_F}\right)}{\left(\frac{1}{R_o}\right)\left(\frac{1}{R_F}\right)}$$

$$= 1 + \frac{R_F}{R_I}$$

which is the ideal gain equation.

4.2 The network in Fig. 4.2 can be reduced to that shown in Fig. S4.2(a) by combining resistors.

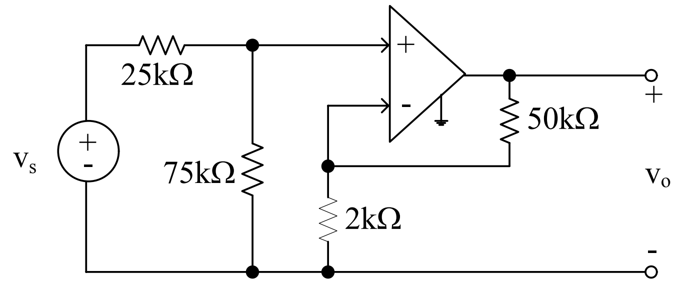

Fig. S4.2(a)

v_+ is determined by the voltage divider at the input, i.e.

$$v_+ = v_s\left[\frac{75k}{25k + 75k}\right] = \frac{3}{4}v_s$$

The op-amp is in a standard noninverting configuration and the gain is $1 + {}^{50k}\!/_{2k} = 26$.

Therefore

$$v_o = (26)\left({}^{3}\!/_{4}\, v_s\right)$$

and

$$\frac{v_o}{v_s} = 19.5$$

4.3 The node equations for the circuit in Fig. 4.3 are

$$\frac{v_s - v_-}{R} + \frac{v_o - v_-}{R + \Delta R} = 0$$

$$\frac{v_s - v_+}{R} = \frac{v_+}{R}$$

$$v_- = v_+$$

Then

$$v_- = v_+ = \frac{v_s}{2}$$

$$\frac{v_s - \frac{1}{2}v_s}{R} + \frac{v_o - \frac{1}{2}v_s}{R + \Delta R} = 0$$

$$\frac{v_s}{2R} + \frac{v_o}{R + \Delta R} - \frac{v_s}{2(R + \Delta R)} = 0$$

$$\frac{v_o}{R + \Delta R} = v_s \left[\frac{1}{2(R + \Delta R)} - \frac{1}{2R} \right]$$

$$= v_s \left[\frac{-\Delta R}{(2R)(R + \Delta R)} \right]$$

$$v_o = v_s \left[\frac{-\Delta R}{2R} \right]$$

$$\frac{v_o}{v_s} = \frac{-\Delta R}{2R}$$

4.4 A weighted-summer circuit shown in Fig. S4.4(a) can be used to produce an output of the form $v_o = -\frac{R}{R_1}v_1 - \frac{R}{R_2}v_2$.

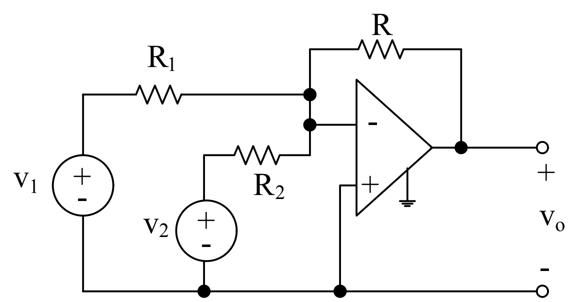

Fig. S4.4(a)

Note that

$$\frac{R}{R_1} = 2 \quad \text{and} \quad \frac{R}{R_2} = \frac{1}{2}$$

Therefore if

$R = 24k\Omega$ (two $12k\Omega$ resistors in series)

$R_1 = 12k\Omega$

$R_2 = 48k\Omega$ (four $12k\Omega$ resistors in series)

then the design conditions are satisfied.

CHAPTER 5 PROBLEMS

5.1 Find V_0 in the circuit in Fig. 5.1 using the Principle of Superposition.

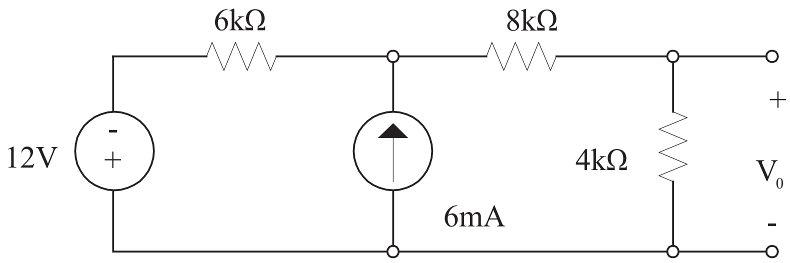

Fig. 5.1

5.2 Solve problem 5.1 using source transformation.

5.3. Find V_0 in the network in Fig. 5.3 using Thevenin's Theorem.

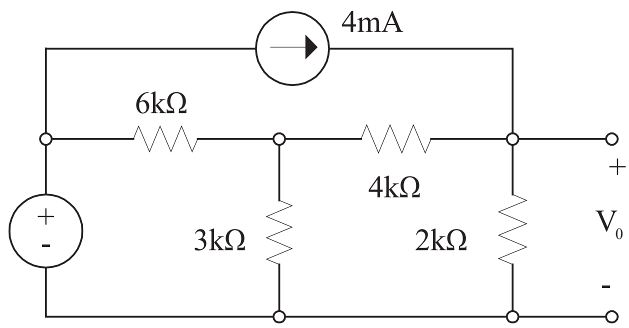

Fig. 5.3

5.4 Find I_0 in the circuit in Fig. 5.4 using Norton's Theorem.

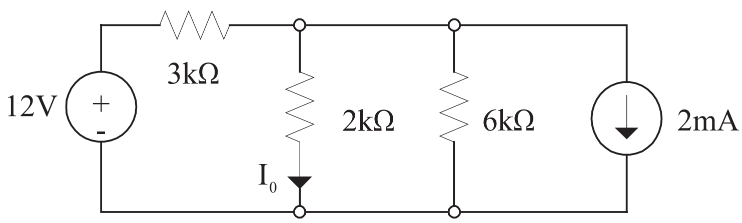

Fig. 5.4

5.5 For the network in Fig. 5.5, find R_L for maximum power transfer and the maximum power that can be transferred to this load.

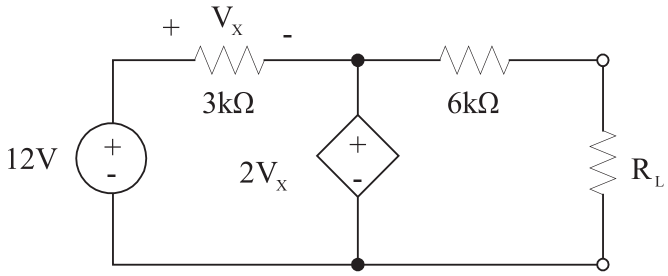

Fig. 5.5

CHAPTER 5 SOLUTIONS

5.1 To apply superposition, we consider the contribution that each source independently makes to the output voltage V_0. In so doing, we consider each source operating alone and we zero the other source(s). Recall, that in order to zero a voltage source, we replace it with a short circuit since the voltage across a short circuit is zero. In addition, in order to zero a current source, we replace the current source with an open circuit since there is no current in an open circuit.

Consider now the voltage source acting alone. The network used to obtain this contribution to the output V_0 is shown in Fig. S5.1(a).

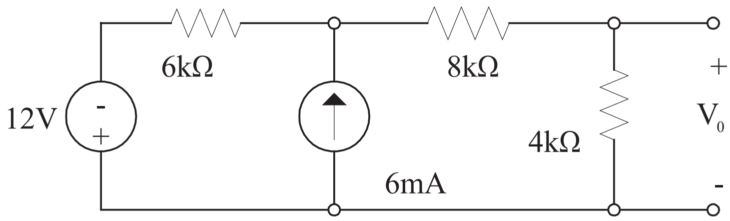

Fig. S5.1(a)

Then V_0' (only a part of V_0) is the contribution due to the 12V source. Using voltage division

$$V_0 = -12\left(\frac{4k}{4k + 6k + 8k}\right)$$
$$= \frac{-8}{3}V$$

The current source's contribution to V_0 is obtained from the network in Fig. S5.1(b).

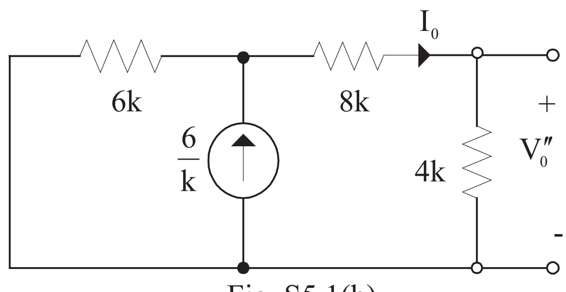

Fig. S5.1(b)

Using current division, we find that

$$I_0 = \frac{6}{k}\left(\frac{6k}{6k + 8k + 4k}\right)$$
$$= 2mA$$

Then

$$V_0'' = 4kI_0$$
$$= 8V$$

Then superposition states that

$$V_0 = V_0' + V_0''$$
$$= \frac{-8}{3} + 8 = \frac{16}{3}V$$

5.2 Recall that when employing source transformation, at a pair of terminals we can exchange a voltage source V_S in series with a resistor R_S for a current source I_p in parallel with a resistor R_p and vice versa, provided that the following relationships among the parameters exist.

$$I_p = \frac{V_S}{R_S}$$
$$R_p = R_S$$

Now the original circuit is shown in Fig. S5.2(a).

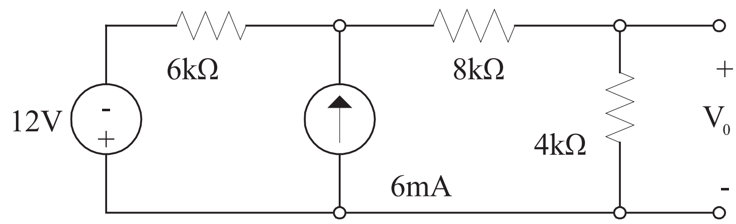

Fig. S5.2(a)

Note that we have a 12V source in series with a 6kΩ resistor that can be exchanged for a current source in parallel with the resistor. This appears to be a viable exchange since we will then have two current sources in parallel which we can add algebraically. Performing the exchange yields the network in Fig. S5.2(b).

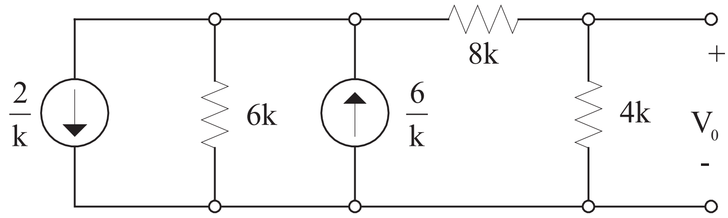

Fig. S5.2(b)

Note that the voltage source was positive at the bottom terminal and therefore the current source points in that direction. Adding the two parallel current sources reduces the network to that shown in Fig. S5.2(c).

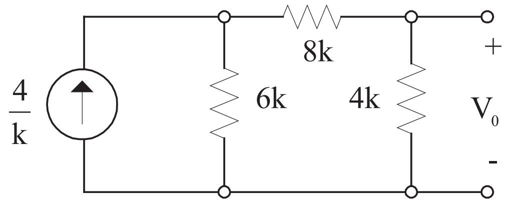

Fig. S5.2(c)

At this point we can apply current division to obtain a solution. For example, the current in the 4kΩ resistor is

$$\begin{aligned} I_{4k} &= \frac{4}{k}\left(\frac{6k}{6k + 8k + 4k}\right) \\ &= \frac{4}{3}\,\text{mA} \end{aligned}$$

Then

$$\begin{aligned} V_0 &= \left(I_{4k}\right)\left(4k\right) \\ &= \frac{16}{3}\,\text{V} \end{aligned}$$

However, we could also transform the current source and the parallel 6kΩ resistor into a voltage source in series with the 6kΩ resistor before completing the solution. If we make this exchange, then the network becomes that shown in Fig. S5.2(d).

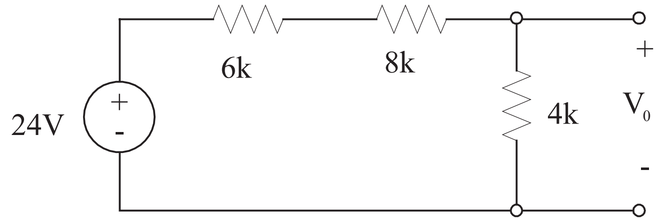

Fig. S5.2(d)

Then using voltage division

$$\begin{aligned} V_0 &= 24\left(\frac{4k}{4k + 6k + 8k}\right) \\ &= \frac{16}{3}\,\text{V} \end{aligned}$$

5.3 Since the network contains no dependent source, we will simply determine the open circuit voltage, V_{0c}, and with the sources in the network made zero, we will look into the open circuit terminals and compute the resistance at these terminals, R_{TH}. The open circuit voltage is determined from the network in Fig. S5.3(a).

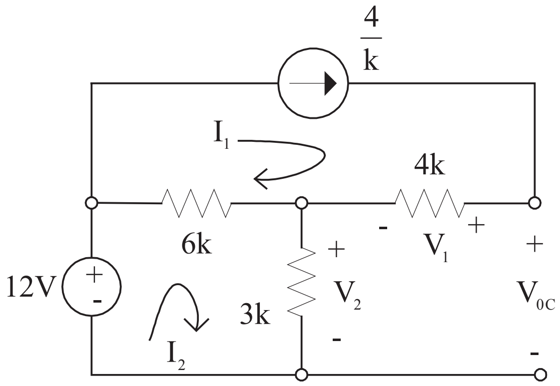

Fig. S5.3(a)

Note the currents and voltages labeled in the network. First of all, note that

$$V_{0C} = V_1 + V_2$$

Therefore, we need only to determine these voltages. Clearly, the voltage V_1 is

$$V_1 = I_1 (4k) = 16V$$

However, to find V_2 we need I_2. KVL around the loop I_2 yields

$$-12 + 6k\,(I_2 - I_1) + 3kI_2 = 0$$

or

$$-12 + 6k\,(I_2 - \frac{4}{k}) + 3kI_2 = 0$$

$$I_2 = \frac{4}{k} = 4mA$$

Now

$$\begin{aligned} V_{0C} &= V_1 + V_2 \\ &= 4kI_1 + 3kI_2 \\ &= 28V \end{aligned}$$

The Thevenin equivalent resistance is found by zeroing all sources and looking into the open circuit terminals to determine the resistance. The network used for this purpose is shown in Fig. S5.3(b).

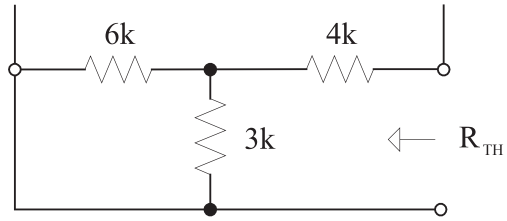

Fig. S5.3(b)

From the network we see that the 6k and 3k Ohm resistors are in parallel and that combination is in series with the 4kΩ resistor. Thus

$$
\begin{aligned}
R_{TH} &= 4k + 3k \,||\, 6k \\
&= 6k\Omega
\end{aligned}
$$

Therefore, the Thevenin equivalent circuit consists of the 28V source in series with the 6kΩ resistor. If we connect the 2kΩ resistor to this equivalent network we obtain the circuit in Fig. S5.3(c).

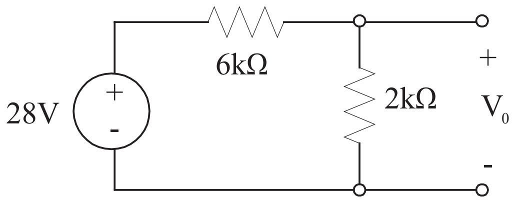

Fig. S5.3(c)

Then using voltage division

$$
\begin{aligned}
V_0 &= 28\left(\frac{2k}{2k + 6k}\right) \\
&= 7V
\end{aligned}
$$

5.4 In this network, the 2kΩ resistor represents the load. In applying Norton's Theorem we will replace the network without the load by a current source, the value of which is equal to the short-circuit current computed from the network in Fig. S5.4(a), in parallel with the Thevenin equivalent resistance determined from Fig. S5.4(b).

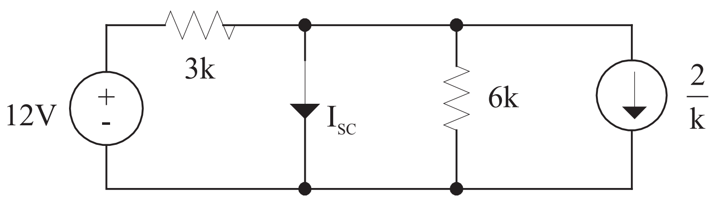

Fig. S5.4(a)

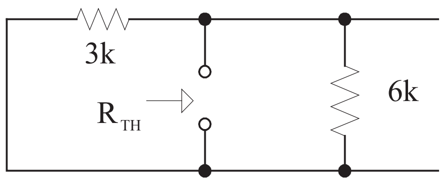

Fig. S5.4(b)

with reference to Fig. S5.4(a), all current emanating from the 12V source will go through the short-circuit. Likewise, all the current in the 2mA current source will also go through the short-circuit so that

$$I_{SC} = \frac{12}{3k} - \frac{2}{k} = 2mA$$

If this statement is not obvious to the reader, then consider the circuit shown in Fig. S5.4(c).

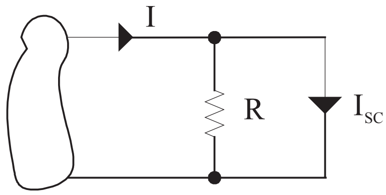

Fig. S5.4(c)

Knowing that the resistance of the short-circuit is zero, we can apply current division to find I_{SC}

$$I_{SC} = I\left(\frac{R}{R+0}\right)$$
$$= I$$

indicating that all the current in this situation will go through the short-circuit and none of it will go through the resistor. From Fig. S5.4(b) we find that the 3k and 6k Ohm resistors are in parallel and thus

$$R_{TH} = 3k \,||\, 6k = 2k\Omega$$

Now the Norton equivalent circuit consists of the short-circuit current in parallel with the Thevenin equivalent resistance as shown in Fig. S5.4(d).

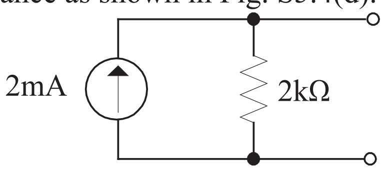

Fig. S5.4(d)

Remember, at the terminals of the 2kΩ load, this network is equivalent to the original network with the load removed. Therefore, if we now connect the load to the Norton equivalent circuit as shown in Fig. S5.4(e), the load current I_0 can be calculated via current division as

$$I_0 = \frac{2}{k}\left(\frac{2k}{2k + 2k}\right)$$
$$= 1 \text{ mA}$$

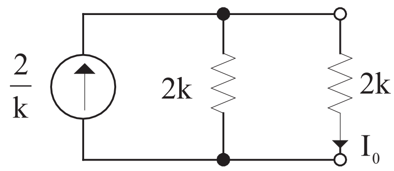

Fig. S5.4(e)

5.5 The solution of this problem involves finding the Thevenin equivalent circuit at the terminals of the load resistor R_L and setting R_L equal to the Thevenin equivalent resistance R_{TH}.

To determine the Thevenin equivalent circuit, we first find the open circuit voltage as shown in Fig. S5.5(a).

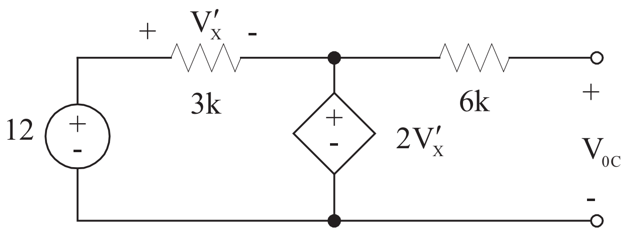

Fig. S5.5(a)

We employ the prime notation on the control variable V_x since the circuit in Fig. S5.5(a) is different than the original network. Applying KVL to the left side of the network yields

$$-12 + V_x' + 2V_x' = 0$$
$$V_x' = 4V$$

Then the open circuit voltage is

$$V_{0C} = 2V_x'$$
$$= 8V$$

since there is no current in the 6kΩ resistor and therefore no voltage drop across it.

Because of the presence of the dependent source we cannot simply look back into the open circuit terminals, with all independent sources made zero, and determine the Thevenin equivalent resistance. We must determine the short-circuit current, I_{SC} and determine R_{TH} from the expression

$$R_{TH} = \frac{V_{0C}}{I_{SC}}$$

I_{SC} is found from the circuit in Fig. S5.5(b).

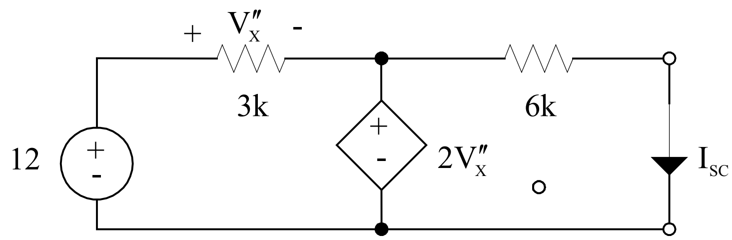

Fig. S5.5(b)

Once again, using KVL

$$-12 + V_x'' + 2V_x'' = 0$$
$$V_x'' = 4$$

Then, since the dependent source $2V_x'' = 8V$ is connected directly across the 6kΩ resistor

$$I_{SC} = \frac{2V_X''}{6k} = \frac{2}{3}\text{mA}$$

and

$$R_{TH} = \frac{V_{0C}}{I_{SC}} = \frac{8}{\frac{2}{3k}} = 12\text{k}\Omega$$

Hence, for maximum power transfer

$$R_L = R_{TH} = 12\text{k}\Omega$$

And the remainder of the problem involves finding the power absorbed by the 12kΩ load, P_L. From the network in Fig. S5.5(c) we find that

$$
\begin{aligned}
P_L &= I_L^2 \, R_L \\
&= \left(\frac{8}{24k}\right)^2 (12k) \\
&= 1.33 \text{ mW}
\end{aligned}
$$

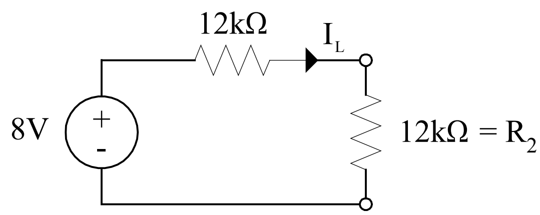

Fig. S5.5(c)

CHAPTER 6 PROBLEMS

6.1 If the voltage across a 10μF capacitor is shown in Fig. 6.1, derive the waveform for the capacitor current.

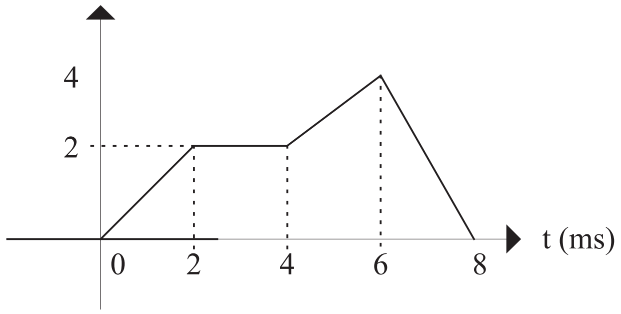

Fig. 6.1

6.2 If the voltage across a 100mH inductor is shown in Fig. 6.2, find the waveform for the inductor current.

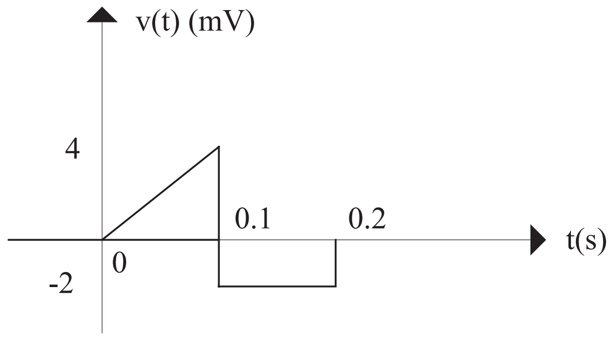

Fig. 6.2

6.3 Find the equivalent capacitance of the network in Fig. 6.3 at the terminals A-B. All capacitors are 6μF.

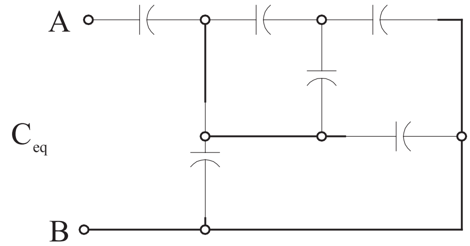

Fig. 6.3

6.4 Find the equivalent inductance of the network in Fig. 6.4 at the terminals A-B. All inductors are 12mH.

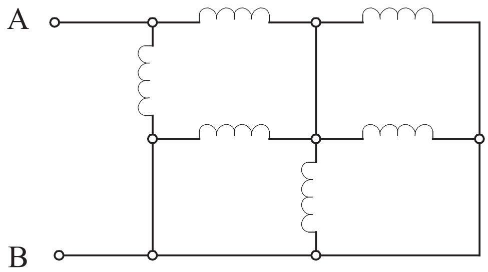

Fig. 6.4

CHAPTER 6 SOLUTIONS

6.1 The equations for the waveforms in the 4 two millisecond time intervals are listed below.

$$
\begin{aligned}
v(t) &= mt + b \\
&= \frac{2}{2\times10^{-3}}\,t && 0 \le t \le 2\text{ms} \\
&= 2 && 2 \le t \le 4\text{ms} \\
&= -2 + \frac{2}{2\times10^{-3}}\,t && 4 \le t \le 6\text{ms} \\
&= +16 - \frac{4}{2\times10^{-3}}\,t && 6 \le t \le 8\text{ms} \\
&= 0 && t < 0,\ \ t > 8\text{ms}
\end{aligned}
$$

Note that within each interval we have simply written the equation of a straight line using the expression y = mx + b or equivalently v(t) = mt + b where m is the slope of the line and b is the point at which the line intersects the v(t) axis.

The equation for the current in a capacitor is

$$i(t) = C\frac{dv(t)}{dt}$$

Using this expression we can compute the current in each interval. For example, in the interval from $0 \le t \le 2\text{ms}$

$$
\begin{aligned}
i(t) &= \left(10\times10^{-6}\right)\frac{d}{dt}\left(\frac{2}{2\times10^{-3}}\,t\right) && 0 \le t \le 2\text{ms} \\
&= 10\text{mA} \\
i(t) &= \left(10\times10^{-6}\right)\frac{d}{dt}(2) && 2 \le t \le 4\text{ms} \\
&= 0 \\
i(t) &= \left(10\times10^{-6}\right)\frac{d}{dt}\left(-2 + \frac{2}{2\times10^{-3}}\,t\right) && 4 \le t \le 6\text{ms} \\
&= 10\text{mA} \\
i(t) &= \left(10\times10^{-6}\right)\frac{d}{dt}\left(16 - \frac{4}{2\times10^{-3}}\,t\right) && 6 \le t \le 8\text{ms} \\
&= -20\text{mA}
\end{aligned}
$$

The waveform for the capacitor current is shown in Fig. S6.1.

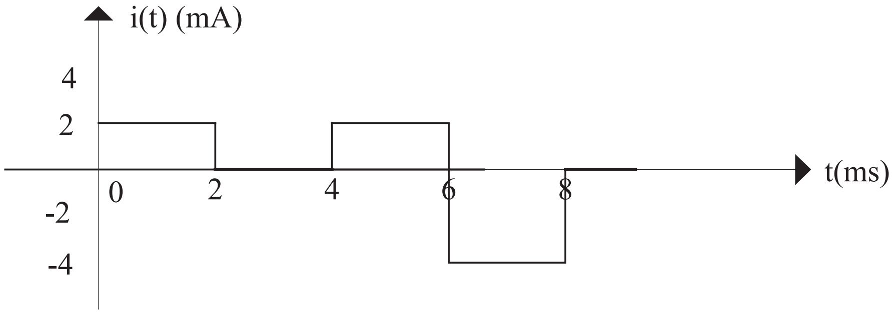

Fig. S6.1

6.2 The general expression for the current in an inductor is

$$i(t) = i(t_0) + \int_{t_0}^{t} v(x)dx$$

In order to evaluate this function we need the equation of the voltage waveform in the two time intervals $0 \le t \le 0.1$s and $0.1 \le t \le 0.2$s. In the first case, the voltage function is a straight line and the function passes through the origin of the graph. The equation of a straight line on this graph is

$$v(t) = mt + b$$

where m is the slope of the line and b is the point at which the line intersects the v(t) axis. Since the slope is $\frac{4 \times 10^{-3}}{0.1}$, the equation of the line is

$$v(t) = \frac{4 \times 10^{-3}}{0.1} t$$

where v(t) is measured in volts and time is measured in seconds i.e., the slope has units of volts/sec. Therefore,

$$i(t) = i(0) + \frac{1}{L}\int_0^t \frac{4 \times 10^{-3}}{0.1} \times dx$$

since there is no initial current in the inductor i(t) = 0 and $\frac{1}{L} = 10$

$$i(t) = 10\int_0^t 4 \times 10^{-2} \times dx$$

or

$$i(t) = 0.4\int_0^t \times\, dx = 0.4\,\frac{x^2}{2}\Big|_0^t$$
$$= 0.2t^2\ \text{A} = 200t^2\ \text{mA}$$

Since the initial current for the second time interval is determined by the value of the current at the end of the first time interval we calculate

$$i(t)\Big|_{t=0.1} = 200t^2\Big|_{t=0.1}\ \text{mA}$$
$$= 2\text{mA}$$

Therefore, in the time interval $0.1 \le t \le 0.2$s

$$i(t) = i(0.1) + \frac{1}{L}\int_{0.1}^{t} v(x)\,dx$$

Note that in this interval v(x) is a constant –2mV or -2×10^{-3}V. Hence,

$$i(t) = 2\times10^{-3} + 10\int_{0.1}^{t}\left(-2\times10^{-3}\right)dx$$
$$= 2\times10^{-3} - 20\times10^{-3} \times\Big|_{0.1}^{t}$$
$$= (4 - 20t)\text{mA}$$

If we now plot the two functions for the current within their respective time intervals we obtain the plot in Fig. S6.2.

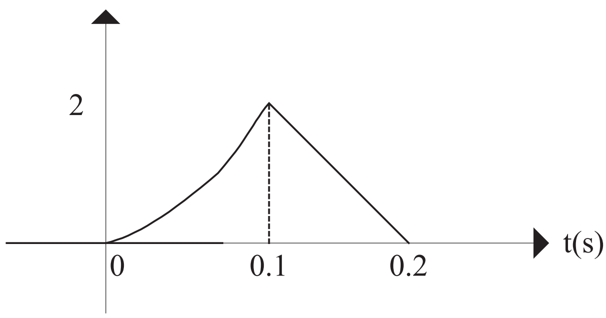

Fig. S6.2

6.3 To begin our analysis we first label all the capacitors and nodes in the network as shown in Fig. S6.3(a).

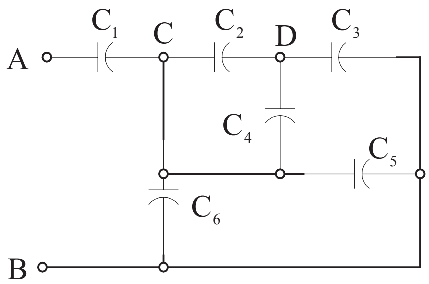

Fig. S6.3(a)

First of all, the reader should note that all the nodes have been labeled, i.e., there are no other nodes. As we examine the topology of the network we find that since C_3 and C_5 are both connected to node D the network can be redrawn as shown in Fig. S6.3(b).

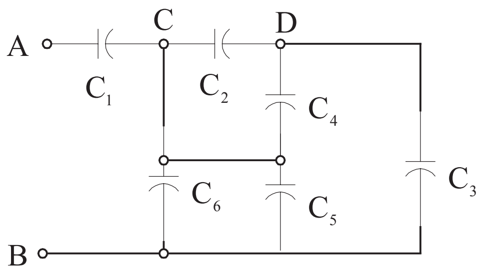

Fig. S6.3(b)

Clearly, C_5 and C_6 are in parallel and their combination we will call $C_{56} = C_5 \,||\, C_6$. Combining these two capacitors reduces the network to that shown in Fig. S6.3(c).

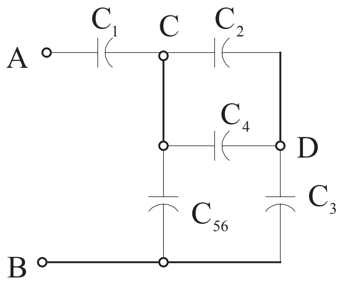

Fig. S6.3(c)

At this point we find that C_2 and C_4 are in parallel and their combination, which we call $C_{24} = C_2 \,||\, C_4$, reduces the network to that shown in Fig. S6.3(d).

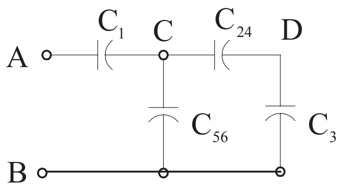

Fig. S6.3(d)

If we now use the given capacitor values, the network becomes that shown in Fig. S6.3(e).

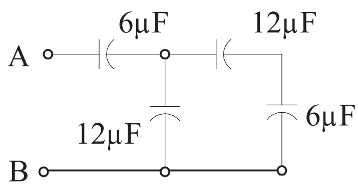

Fig. S6.3(e)

Starting at the opposite end of the network from the terminals A-B and combining elements we find that 6μF in series with 12μF is 4μF and this equivalent capacitance is in parallel with 12μF yielding 16μF, which in turn is in series with 6μF producing a total capacitance of

$$\begin{aligned} C_{eq} &= 6\mu F \| 16\mu F \\ &= 4.36\mu F \end{aligned}$$

6.4 To aid our analysis, we will first label all inductors and nodes as shown in Fig. S6.4(a).

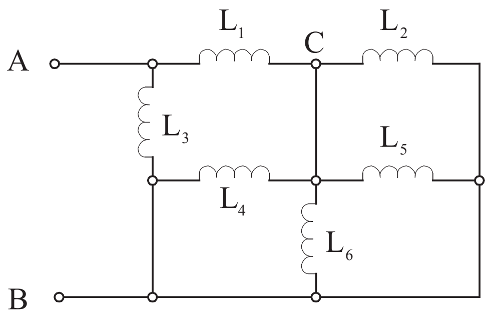

Fig. S6.4(a)

Note carefully that all the nodes have been labeled. Once readers recognize that there are no other nodes, they are well on their way to reducing the network since this node recognition provides data indicating which elements are in series or parallel. For example, since one end of L_4 is connected to node B, the network can be redrawn as shown in Fig. S6.4(b).

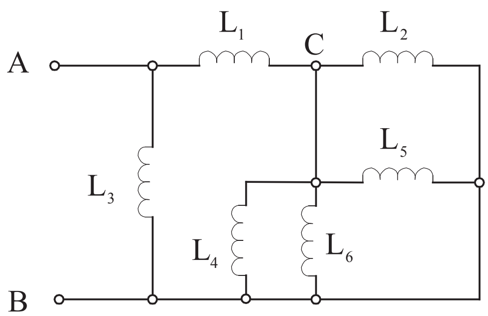

Fig. S6.4(b)

This diagram clearly indicates that L_2 and L_5 are in parallel. In addition, L_4 and L_6 are in parallel. Therefore, if we combine elements so that $L_{25} = L_2 || L_5$ and $L_{46} = L_4 || L_6$, then the circuit can be reduced to that in Fig. S6.4(c).

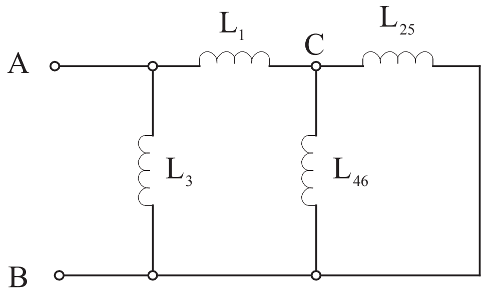

Fig. S6.4(c)

However, we note now if we did not see it earlier that L_{25} is in parallel with L_{46} so that the network can be reduced to that shown in Fig. S6.4(d).

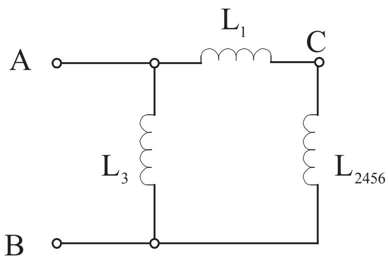

Fig. S6.4(d)

Where $L_{2456} = L_{25} || L_{46}$. Since all inductors are 12mH, L_{2456} = 3mH which is in series with 12mH and that combination is in parallel with 12mH yielding

$$L_{AB} = 12\text{mH} || 15\text{mH} = 6.66\text{mH}$$

CHAPTER 8 PROBLEMS

8.1 Find the frequency domain impedance Z, shown in Fig. 8.1.

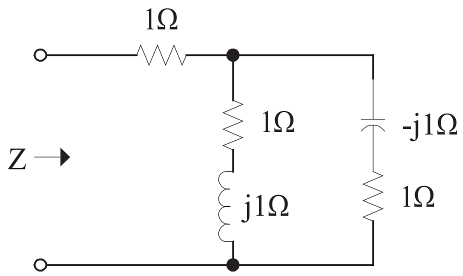

Fig. 8.1

8.2 If the impedance of the network in Fig. 8.2 is real at f = 60Hz, what is the value of the inductor L?

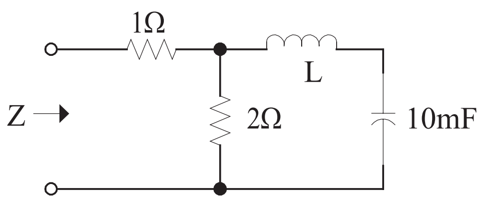

Fig. 8.2

8.3 Use nodal analysis to find V_0 in the network in Fig. 8.3.

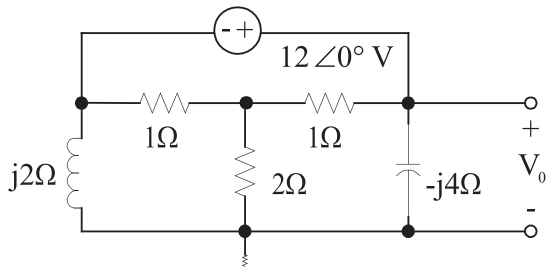

Fig. 8.3

8.4 Find V_0 in the network in Fig. 8.4 using (a) loop analysis (b) superposition and (c) Thevenin's Theorem.

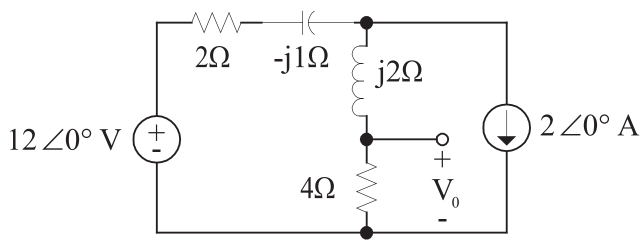

Fig. 8.4

CHAPTER 8 SOLUTIONS

8.1. To begin our analysis, we note that the circuit can be labeled as shown in Fig. S8.1.

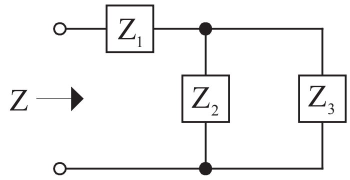

Fig. S8.1

In this case, Z_1 consists of a 1Ω resistor, Z_2 is the series combination of a 1Ω resistor and a j1Ω inductor and Z_2 consists of a –j1Ω capacitor in series with a 1Ω resistor. Therefore,

$$Z_1 = 1\Omega$$
$$Z_2 = 1 + j1\Omega$$
$$Z_3 = 1 - j1\Omega$$

Starting at the opposite end of the network from the terminals at which Z is calculated we note that Z_2 and Z_3 are in parallel and their combination is in series with Z_1. Hence

$$\begin{aligned} Z &= Z_1 + Z_2 \parallel Z_3 \\ &= 1 + \frac{(1+j)(1-j)}{1+j+1-j} \\ &= 1 + \frac{2}{2} \\ &= 2\Omega \end{aligned}$$

8.2 The general expression for the impedance of this network is

$$Z = 1 + 2 \left\| \left(j\omega L + \frac{1}{j\omega C} \right) \right.$$

In order for Z to be purely resistive, the term $\left(j\omega L + \dfrac{1}{j\omega C} \right)$ must be real, i.e.

$$Z_{LC} = R_{LC} + j0$$

However, since Z_{LC} can be written as

$$Z_{LC} = j\left(\omega L - \frac{1}{\omega C} \right)$$

it is clearly an imaginary term and $R_{LC} = 0$. Therefore, in order for Z to be resistive

$$\omega L - \frac{1}{\omega C} = 0$$

or

$$\begin{aligned} L &= \frac{1}{\omega^2 C} \\ &= \frac{1}{(377)^2 (10^{-2})} \\ &= 703.6\,\mu H \end{aligned}$$

8.3 The presence of the voltage source indicates that nodal analysis is a viable approach to this problem. The voltage source and its two connecting nodes form a supernode as shown in Fig. S8.3.

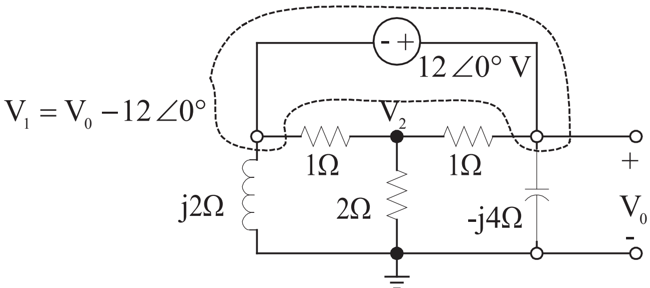

Fig. S8.3

Note that there are three non-reference nodes, i.e., V_1, V_2 and V_0. Because the voltage source is tied directly between nodes V_1 and V_0, $V_1 = V_0 - 12\angle 0°$. This constraint condition is one of our three equations required to solve the network. The two remaining equations are obtained by applying KCL at the supernode and the node labeled V_2. For the supernode, KCL yields

$$\frac{V_1}{j2} + \frac{V_1 - V_2}{1} + \frac{V_0 - V_2}{1} + \frac{V_0}{-j4} = 0$$

At the node labeled V_2, KCL yields

$$\frac{V_2 - V_1}{1} + \frac{V_2}{2} + \frac{V_2 - V_0}{1} = 0$$

Therefore, the three equations that will provide the node voltages are

$$V_1 = V_0 - 12$$

$$-j\frac{1}{2}V_1 + V_1 - V_2 + V_0 - V_2 + j\frac{1}{4}V_0 = 0$$

$$V_2 - V_1 + \frac{1}{2}V_2 + V_2 - V_0 = 0$$

Substituting the first equation in for the two remaining equations and combining terms yields

$$V_0\left(2 - j\frac{1}{4}\right) - 2V_2 = 12 - j6$$

$$-2V_0 + \frac{5}{2}V_2 = -12$$

Solving for V_2 in this last equation and substituting it into the one above it, we obtain

$$V_0(0.4 - j0.25) = 2.4 - j6$$

and hence

$$V_0 = 13.57 \angle -36.2° \text{ V}$$

8.4 (a) Since the network has two loops, or in this case two meshes, we will need two equations to determine all the currents. Consider the network as labeled in Fig. S8.4(a).

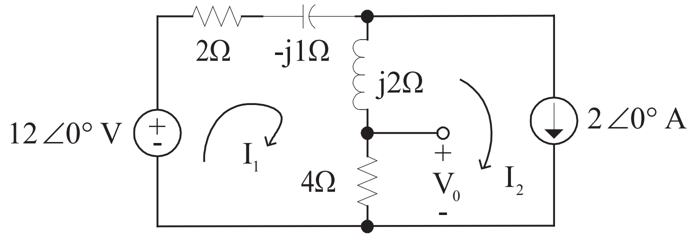

Fig. S8.4(a)

Note that since I_2 goes directly through the current source, I_2 must be $2\angle 0°$A. Hence, one of our two equations is

$$I_2 = 2\angle 0°$$

If we now apply KVL to the loop on the left of the network, we obtain

$$-12 + I_1(2 - j1) + (I_1 - I_2)(4 + j2) = 0$$

These two equations will yield the currents. Substituting the first equation into the second yields

$$-12 + I_1(2 - j1 + 4 + j2) - 2(4 + j2) = 0$$

and then

$$I_1 = \frac{20 + j4}{6 + j1} = 3.35\angle 1.85° \text{ A}$$

Finally,

$$\begin{aligned} V_0 &= 4(I_1 - I_2) \\ &= 4\left(\frac{20 + j4}{6 + j1} - 2\right) \\ &= 5.42\angle 4.57° \text{ V} \end{aligned}$$

(b) In applying superposition to this problem, we consider each source acting alone. If we zero the current source, i.e., replace it with an open circuit, the circuit we obtain is shown in Fig. S8.4(b).

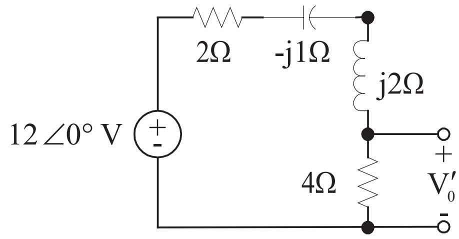

Fig. S8.4(b)

Using voltage division

$$\begin{aligned} V_0' &= 12\left(\frac{4}{4 + j2 + 2 - j1}\right) \\ &= \frac{48}{6 + j1} \text{V} \end{aligned}$$

Now, if we zero the voltage source, i.e., replace it with a short circuit, we obtain the circuit in Fig. S8.4(c).

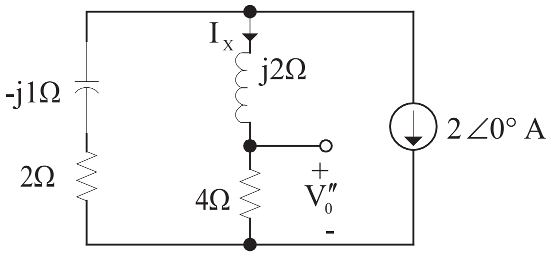

Fig. S8.4(c)

Employing current division, the current I_X is

$$I_X = -2\angle 0°\left(\frac{2-j}{2-j+4+j2}\right)$$
$$= \frac{-4+j2}{6+j1}\ \text{A}$$

Then,

$$V_0'' = 4I_X = \frac{-16+j8}{6+j1}$$

And finally,

$$V_0 = V_0' + V_0''$$
$$= \frac{48}{6+j1} + \frac{-16+j8}{6+j1}$$
$$= \frac{32+j8}{6+j1}$$
$$= 5.42\angle 4.57°\ \text{V}$$

(c) In applying Thevenin's Theorem, we first break the network at the load and determine the open-circuit voltage as shown in Fig. S8.4(d).

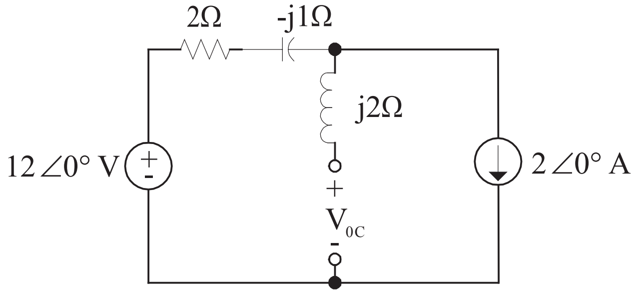

Fig. S8.4(d)

Note that there exists only one closed path and the current in it must be $2\angle 0°$ A. Note also that there is no current in the inductor and therefore no voltage across it. Hence V_{OC} is also the voltage across the current source. Hence,

$$\begin{aligned} V_{0C} &= 12 - 2(2 - j) \\ &= 8 + j2 \text{ V} \end{aligned}$$

The Thevenin equivalent impedance found by zeroing the independent sources and looking into the network at the terminals of the load can be determined from the circuit in Fig. S8.4(e).

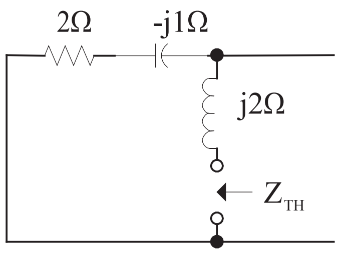

Fig. S8.4(e)

This network indicates that

$$\begin{aligned} Z_{TH} &= 2 - j1 + j2 \\ &= 2 + j1\Omega \end{aligned}$$

If we now form the Thevenin equivalent circuit and re-connect the load, we obtain the network in Fig. S8.4(f).

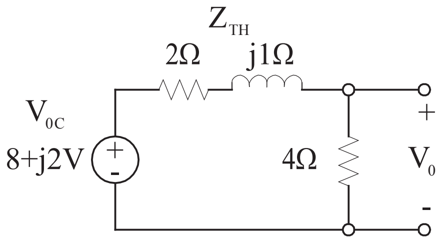

Fig. S8.4(f)

Applying voltage division yields

$$\begin{aligned} V_0 &= (8 + j2)\left(\frac{4}{4 + 2 + j1}\right) \\ &= \frac{32 + j8}{6 + j1} \\ &= 5.42\angle 4.57° \text{ V} \end{aligned}$$

CHAPTER 9 PROBLEMS

9.1 Determine the average power supplied by each source in the circuit in Fig. 9.1.

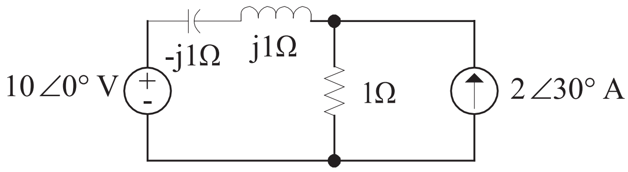

Fig. 9.1

9.2 Given the circuit in Fig. 9.2, determine the impedance Z_L for maximum average power transfer and the value of the maximum average power transferred to this load.

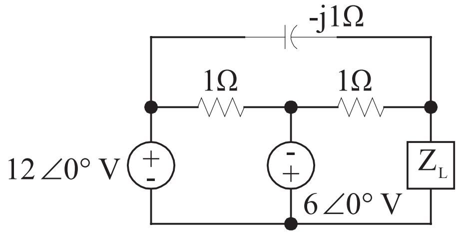

Fig. 9.2

9.3 Calculate the rms value of the waveform shown in Fig. 9.3.

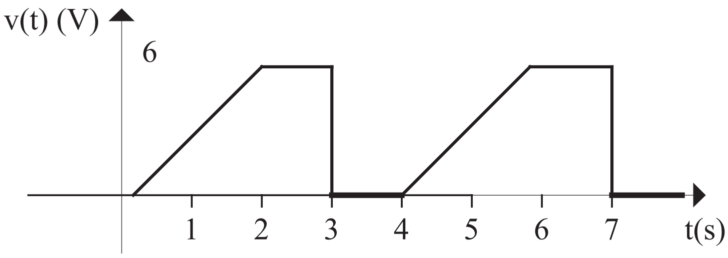

Fig. 9.3

9.4 Determine the source voltage in the network shown in Fig. 9.4.

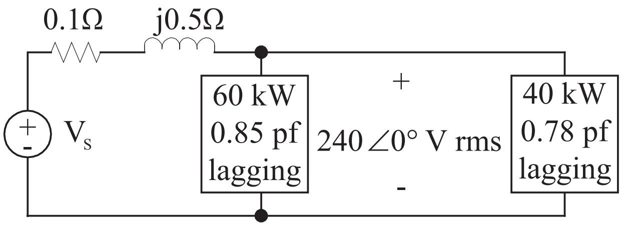

Fig. 9.4

9.5 A plant consumes 75 kW at a power factor of 0.70 lagging from a 240-V rms 60 Hz line. Determine the value of the capacitor that when placed in parallel with the load will change the load power factor to 0.9 lagging.

CHAPTER 9 SOLUTIONS

9.1 Because the series impedance of the inductor and capacitor are equal in magnitude and opposite in sign, from the standpoint of calculating average power the network can be reduced to that shown in Fig. S9.1.

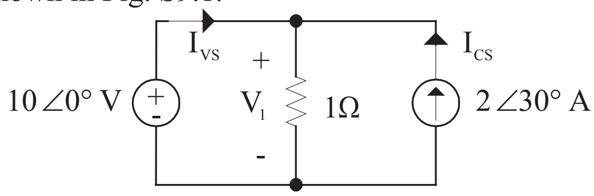

Fig. S9.1

The general expression for average power is

$$P = \frac{1}{2} VI \cos(\theta_V - \theta_I)$$

In the case of the current source $V_1 = 10V$, $I_{CS} = 2A$, $\theta_V = 0°$ and $\theta_I = 30°$. Therefore, the average power delivered by the current source is

$$P_{CS} = \left(\frac{1}{2}\right)(10)(2)\cos(-30°)$$
$$= 8.66 \text{ W}$$

In order to calculate the average power delivered by the voltage source, we need the current I_{VS}. Using KCL

$$I_{VS} + 2\angle 30° = \frac{V}{1} = 10\angle 0°$$

or

$$I_{VS} = 8.33\angle{-6.9°} \text{ A}$$

Now

$$P_{VS} = \frac{1}{2}(10)(8.33)\cos(0° - (-6.9°))$$
$$= 41.34 \text{ W}$$

Therefore, the total power generated in the network is

$$P_T = P_{CS} + P_{VS}$$
$$= 50 \text{ W}$$

Let us now calculate the average power absorbed by the resistor. We know that the average power absorbed by the resistor must be

$$\begin{aligned} P_R &= \frac{1}{2}\frac{V_m^2}{R} \\ &= \frac{1}{2}\left(\frac{10^2}{1}\right) \\ &= 50 \text{ W} \end{aligned}$$

In addition, the average power absorbed by the resistor can also be determined by

$$P_R = \frac{1}{2} I_m^2 R$$

However, we do not know the current in the resistor. Using KCL.

$$\begin{aligned} I_R &= I_{VS} + I_{CS} \\ &= 8.66\angle{-6.9°} + 2\angle 30° \\ &= 10\angle 0° \text{ A} \end{aligned}$$

Now

$$\begin{aligned} P_R &= \frac{1}{2}(10)^2(1) \\ &= 50 \text{ W} \end{aligned}$$

Thus, we find that the total average power generated is equal to the average power absorbed.

9.2 We will first determine the Thevenin equivalent circuit for the network without the load attached. The open-circuit voltage, V_{0C}, can be determined from the network in Fig. S9.2(a).

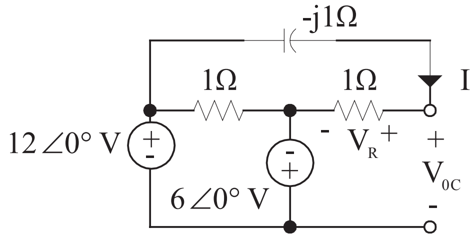

Fig. S9.2(a)

This open-circuit voltage can be calculated in a number of ways. For example, we can compute the current I as

$$I = \frac{12\left(0° - \left(-6\angle 0°\right)\right)}{1-j} = \frac{18}{1-j}\ \mathrm{A}$$

Then using KVL,

$$\begin{aligned} V_{0C} &= 1I - 6\angle 0° \\ &= \frac{12+6j}{1-j}\ \mathrm{V} \end{aligned}$$

or, we could use voltage division to determine the voltage across the 1-Ohm resistor on the right, i.e.,

$$\begin{aligned} V_R &= \left[12\angle 0° - \left(-6\angle 0°\right)\right]\left(\frac{1}{1-j}\right) \\ &= \frac{18}{1-j}\ \mathrm{V} \end{aligned}$$

Then, once again

$$\begin{aligned} V_{0C} &= V_R - 6\angle 0° \\ &= \frac{12+6j}{1-j}\ \mathrm{V} \\ &= 9.49\angle 71.56°\ \mathrm{V} \end{aligned}$$

The Thevenin equivalent impedance is obtained by looking into the open-circuit terminals with all sources made zero. In this case, we replace the voltage sources with short circuits. This network is shown in Fig. S9.2(b).

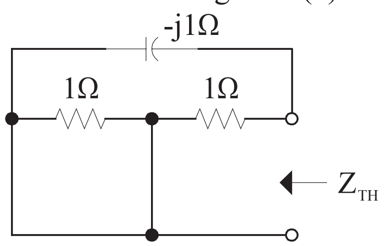

Fig. S9.2(b)

Note that the 1-Ohm resistor on the left is shorted and thus the Z_{TH} is

$$\begin{aligned} Z_{TH} &= \frac{(1)(-j)}{1-j} = \frac{-j}{1-j}\ \Omega \\ &= \frac{1}{2} - j\frac{1}{2}\ \Omega \end{aligned}$$

Hence, for maximum average power transfer

$$Z_L = Z_{TH}^*$$

or

$$Z_L = \frac{1}{2} + j\frac{1}{2}\,\Omega$$

Therefore, the network is reduced to that shown in Fig. S9.2(c).

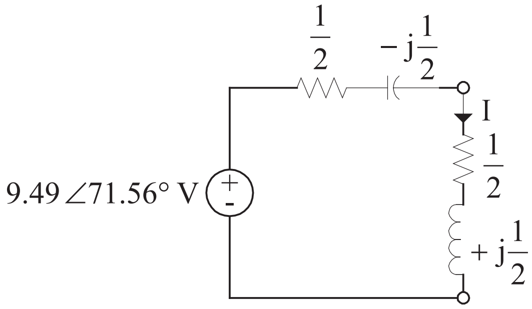

Fig. S9.2(c)

Then

$$I = \frac{9.49\angle 71.56^\circ}{\frac{1}{2} - j\frac{1}{2} + \frac{1}{2} + j\frac{1}{2}}$$
$$= 9.49\angle 71.56^\circ \text{ A}$$

and the maximum average power transferred to the load is

$$P_L = \frac{1}{2}(9.49)^2\left(\frac{1}{2}\right)$$
$$= 90 \text{ W}$$

9.3 In order to calculate the rms value of the waveform, we need the equations for the waveform within each of the distinctive intervals.

In the interval $0 \le t \le 2$s, the waveform is a straight line that passes through the origin of the graph. The equation for a straight line in this graph is

$$v(t) = mt + b$$

Where m is the slope of the line and b is the v(t) intercept. Since the line goes through the origin, b = 0. The slope m is

$$m = \frac{6V}{2s} = 3$$

Therefore, in the interval $0 \leq t \leq 2s$,

$$v(t) = 3t$$

The waveform has constant values in the intervals $2 \leq t \leq 3s$ and $3 \leq t \leq 4s$, i.e.,

$$\begin{aligned} v(t) &= 6 \qquad 2 \leq t \leq 3s \\ v(t) &= 0 \qquad 3 \leq t \leq 4s \end{aligned}$$

Since the waveform repeats after 4s, the period of the waveform is

$$T = 4s$$

Now that the data for the waveform is known,

$$V_{rms} = \left[\frac{1}{T}\int_0^4 v^2(t)\,dt\right]^{\frac{1}{2}}$$

Therefore, in this case

$$\begin{aligned} V_{rms} &= \left[\frac{1}{4}\left[\int_0^2 (3t)^2\,dt + \int_2^3 (6)^2\,dt + \int_3^4 (0)^2\,dt\right]\right]^{\frac{1}{2}} \\ &= \left[\frac{1}{4}\left[3t^3\Big|_0^2 + 36t\Big|_2^3\right]\right]^{\frac{1}{2}} \\ &= \left[\frac{1}{4}(24+36)\right]^{\frac{1}{2}} \\ &= (15)^{\frac{1}{2}} \\ &= 3.87 \text{ V rms} \end{aligned}$$

9.4 We begin our analysis by labeling the various currents and voltages in the circuit as shown in Fig. S9.4.

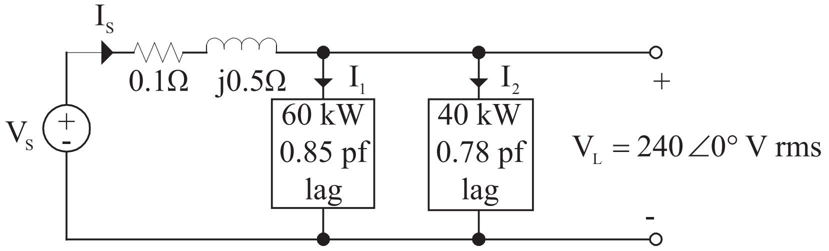

Fig. S9.4

Our approach to determining V_S is straight forward: We will compute the currents I_1 and I_2; add them using KCL to find I_S; determine the voltage across the line impedance and finally use KVL to add the line voltage and load voltage to determine the source voltage.

The magnitude of the current I_1 is

$$\begin{aligned}|I_1| &= \frac{P_1}{|V_L|(pf_1)}\\ &= \frac{60{,}000}{(240)(0.85)}\\ &= 294.12 \text{ A rms.}\end{aligned}$$

And the phase angle is

$$\begin{aligned}\theta_{I_1} &= -\cos^{-1}(0.85)\\ &= -31.79°\end{aligned}$$

The negative sign is a result of the fact that the power factor is lagging.

Thus

$$I_1 = 294.12\angle\text{-}31.79° \text{ A rms.}$$

The magnitude of the current I_2 is

$$\begin{aligned}|I_2| &= \frac{P_2}{|V_L|(pf_2)}\\ &= \frac{40{,}000}{(240)(0.78)}\\ &= 213.68 \text{ A rms.}\end{aligned}$$

And the phase angle is

$$\begin{aligned}\theta_{I_2} &= -\cos^{-1}(0.78)\\ &= -38.74°\end{aligned}$$

Thus

$$I_2 = 213.68\angle\text{-}38.74° \text{ A rms.}$$

Using KCL

$$\begin{aligned} I_S &= I_1 + I_2 \\ &= 294.12\angle -31.79° + 213.68\angle -38.74° \\ &= 504.1\angle -34.25° \text{ A rms.} \end{aligned}$$

Then

$$\begin{aligned} V_2 &= I_S\,(0.1 + j0.5) + 240\angle 0° \\ &= (504.1\angle -34.25°)(0.51\angle 78.7°) + 240\angle 0° \\ &= 257.04\angle 44.44° + 240\angle 0° \\ &= 460.17\angle 23.02° \text{ V rms.} \end{aligned}$$

9.5 Since the original power factor is 0.7 lagging the power factor angle is

$$\begin{aligned} \theta_{OLD} &= \cos^{-1}(0.7) \\ &= 45.57° \end{aligned}$$

Then

$$\begin{aligned} Q_{OLD} &= P_{OLD} \tan \theta_{OLD} \\ &= 75{,}000 \tan 45.57° \\ &= 76.52 \text{ kvar} \end{aligned}$$

Hence

$$\begin{aligned} S_{OLD} &= 75{,}000 + j76{,}515 \\ &= 107.14\angle 45.57° \text{ kVA} \end{aligned}$$

The new power factor angle we wish to achieve is

$$\begin{aligned} \theta_{NEW} &= \cos^{-1}(\text{new power factor}) \\ &= \cos^{-1}(0.9) \\ &= 25.84° \end{aligned}$$

Then

$$\begin{aligned} Q_{NEW} &= P_{OLD} \tan \theta_{NEW} \\ &= 75{,}000 \tan 25.84° \\ &= 36{,}324 \text{ kvar} \end{aligned}$$

Now the difference between Q_{NEW} and Q_{OLD} is achieved by the capacitor, i.e.,

$$\begin{aligned} Q_{CAP} &= Q_{NEW} - Q_{OLD} \\ &= 36{,}324 - 76{,}515 \\ &= -40{,}191 \text{ kvar} \end{aligned}$$

And since

$$Q_{CAP} = -\omega\, CV^2$$

Then

$$C = \frac{40{,}191}{(377)(240)^2}$$
$$= 1850.8\,\mu F$$