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FRACTAL INTERPOLATION OF GEOCHEMICAL DATA: AN IMPROVED METHOD FOR GEOCHEMICAL TRAVERSE CONSTRUCTION

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ABSTRACT

Algorithms for data interpolation are a major problem in geochemical characterization at big scales, particularly experienced during ...evaluation of prospecting results (based either in soil or rock analysis).

Difficulties related to sampling (lack of outcropping rocks or non accessible areas for soil collection) are two of many factors that ...ously constrain the results obtained by the usual methods of data interpolation.

The presence of a high background noise compromises often the interpretations, generating additional problem on the identification ...retation of the most significant geochemical halos. Notorious differences between the real geometry presented by the geochemical ...ures and the images produced via data interpolation (using more or less sophisticated statistical procedures), are also quite common.

Fractal interpolation has proved to be a powerful tool in problems of highly complex data interpolation, such as the brownian motion.

The application of IFS (Iterated Function Systems) algorithm to sampled data from a theoretical model pointed two main conclusions:

1) The complexity of the interpolated curves closely resembles the complexity of the original data distribution;

2) Geochemical anomalies masked by an unaccurated sampling method were evidenced by the fractal interpolation methodology.

INTRODUCTION

The interpretation of geochemical data is frequently complicated by the discrete and ...metimes inadequate sampling, which allows a quick determination of the general ...aracteristics of the distributions statistics, although the details of such distributions are ...ssed.

The interpolation between sampled data points as always been a major concern to ... geoscientists dealing with chemical elements distribution. The use of linear ...erpolation algorithms was the first attempt to describe the characteristics of a ...pulation from a restricted sample set. The inadequacy of such procedure is evident ...hen we attempt to make data interpolation to sample sets withdrawn from a harmonic ...nction for example. This led to the use of increasingly complex functions in interpolation ...orithms such as polynomial or harmonic. Although the use of these functions is ...siderably more complex than the simple linear interpolation procedure, their ability to ...scribe highly irregular objects is very restricted since the degree of complexity is ...ngly dependent on the observation scale.

During the last two decades the theoretical development of fractal geometry ...epts evidenced a wide group of new functions, with infinite degree of complexity, ... describe in a suitable way natural forms and processes. These new functions are ...werful tools to solve problems of data interpolation in highly complex populations.

The pattern of chemical elements distribution in the crust is mainly constrained by ... outcropping lithologies; 2) sampling procedures; and 3) intrinsic analytical errors. ...sequently, for most of the geological contexts, a geochemical traverse is an irregular ... and so an interpolation method based on fractal geometry is probably much more ...propriate to characterize it than any other statistical approach.

ITERATED FUNCTION SYSTEMS (IFS)

There are several ways to construct a fractal object (for more details see Peitgen al. 1992), although Iterated Function Systems (IFS) are one of the simplest for drawing deterministic attractors (such as the Von Koch curve, the Sierpinski carpet or t Merenger sponge) through the estimation of **y** values from randomly selected **x** value The method is based on the application of randomly selected at each iteration line transformations W_i, which are combinations of contractions, rotations, shears a translations.

The deterministic attractor has a number of fixed points controlling its geomet which verify:

let F: $X \rightarrow X$ be a transformation in the metric space.

If $x \in X$ and $F(x)=x$ then x is a fixed point.

The linear transformations W_i may be calculated from the attractor fixed points a have the general formula:

$$W_i \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a_i & b_i \\ c_i & d_i \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} e_i \\ f_i \end{bmatrix} \qquad (1)$$

where a_i, b_i, c_i, d_i are parameters that control contractions, shears and rotations (and ha values ranging from -1 to 1) while e_i and f_i describe the translations.

During the first iteration a transformation W_i is randomly selected among the all s of possible transformations, and applied to the initial point (x_0,y_0), giving as result a ne point belonging to the attractor (x_1,y_1). In the second iteration a new random selection W_i is made and then applied to (x_1,y_1) giving (x_2,y_2) as a result, and so forward. Th number of iterations needed to define the attractor depends on the contractivity of th fractal image (Horn, 1991).

FRACTAL INTERPOLATION

Barnsley (1988) proposed an IFS based method capable of interpolate bright complex functions as fractal curves. Using data points as fixed points (since th interpolated curve must contain them) the data related transformations can be determine and then used to construct an interpolated curve. In order to simplify the number o unknown parameters, Barnsley (1988) constrained the shears along X by setting b zero. With this simplification the general formula of transformations becomes:

$$W_i \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a_i & 0 \\ c_i & d_i \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} e_i \\ f_i \end{bmatrix} \qquad (2)$$

Since in this new formulation there are more parameters than linear equations, Barnsley left d_i as free parameters, estimated by the user for each case. However, for the determination of $\mathbf{d_i}$, the above mentioned procedure as two main disadvantages:

1) for N data points there will be N-1 **d** parameters, making individual assessment very time consuming;

2) the evaluation by the user strongly biases the final interpolation result, because this can only be done if there is a previous expectation concerning the results.

Efforts have been made to estimate $\mathbf{d}_i$ parameters directly from the data and although encouraging results were achieved the basic postulate of $\mathbf{d} \in [-1,1]$ was not respected (for more details see Demko, 1989).

In the present work the basic principles of fractal interpolation of geochemical data were applied, namely for the construction of geochemical traverses. A computing routine capable of performing fractal interpolation of two-dimensional data was developed. The lack of continuity of natural data due to poor outcropping or economic constrains led us to the construction of a theoretical model which represents a gold-vein mineralization with values strongly detached from the background. From a 400x400 points grid several geochemical traverses were constructed, from which sets of 40 samples were randomly selected. The routine developed was then applied to the 40 sample sets, and the results were compared with the complete traverses. Since the attractor construction is strongly dependent on $\mathbf{d}_i$ parameters several attempts were made to devise a method of determining them from the data sets.

The formulation proposed by Barnsley (1988) has another disadvantage when trying to interpolate natural data since this author used all the sampled data simultaneously for the calculation of W_i. This option is supported by the concept of self-similarity characteristic of all fractal objects, in which the geometry of every piece must reflect the geometry of the whole object. However geological objects are not strictly self-similar, and the studied object - gold-vein mineralization - at least two reasons may be pointed questioning the existence of strict self-similarity: 1) the primary concentrations of gold occur in discrete areas with a restricted scale of characterization; 2) results from fractal modelling studies of gold contents in drill-cores (Ribeiro, 1994), suggests that fractal behavior of gold concentrations distribution is only valid in specific scale ranges. Therefore the determination of W_i must be done between data points with a significant correlation and not between all data points as proposed by Barnsley (1988).

RESULTS AND DISCUSSION

In order to evaluate the potential of fractal interpolation procedures, the developed algorithm was applied to the following different cases:

- data sampled from Euclidean functions (e.g. straight line, sine curve, parabola);
- data sampled from the host-rocks of the theoretic mineralization (background values only);
- data sampled from the theoretic model of a gold-vein mineralization.

In cases 2) and 3) a linear interpolation algorithm was also applied to the sampled data enhanced the main differences between the two methods.

Several attempts were made in order to properly estimate $\mathbf{d}_i$ parameters, and the most promising considers a proportionality relation between $\mathbf{d}$ and the sine of the angle formed by the vectors connecting data points to the axis origin.

Fractal interpolation curve, linear interpolation curve and the complete traverse curve cross cutting the ore (fig. 1) were compared. The following conclusions were reached:

i) linear interpolation produces an oversimplified image of the complete geochemical traverse masking the true complexity of the curve;

ii) the fractal interpolation algorithm produces more complex curves (closer to the original population curve) and geochemical anomalies in fig. 1 (arrow), masked by linear interpolation, are evidenced by this interpolation method with a anomaly/background ratio closer to the reality.

iii) some discrepancies between the fractal interpolation curves and the original data curves, are due to inaccurate evaluation of **d** parameters (controlling shear along **Y**).

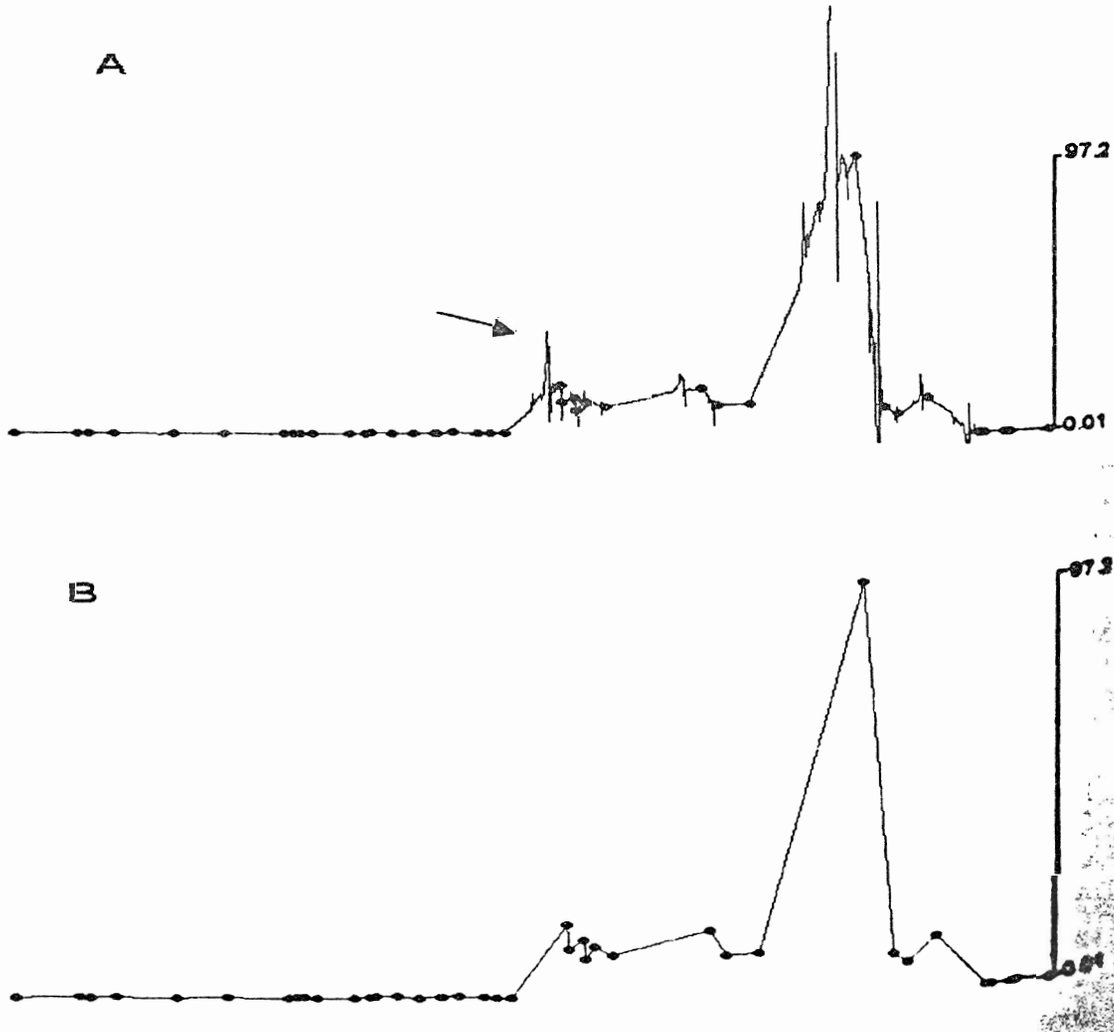
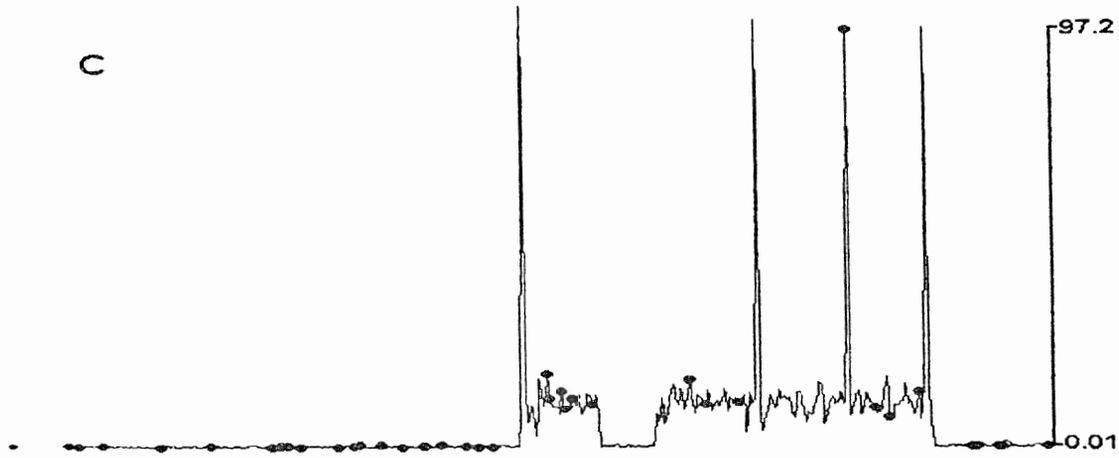


Fig. 1 - Geochemical traverses through the ore. A: Fractal interpolation results; B: Linear interpolation results. C [illegible] traverse. Circles represent sample location. Vertical scaling in ppm. Horizontal axis representing distance from the [illegible] origin.

CONCLUSIONS

The interpolation of data from highly complex populations has always been a major problem in science. The use of interpolant functions from Euclidean geometry was a poor and sometimes not satisfying approach to data handling. Geochemical traverses are among the group of very complex curves, due to the intrinsic variability of elements distribution, poor sampling and analytical constrains. Usually, in geochemical surveys the traverse construction is made by linear interpolation of the data, that although describing general characteristics of the population distribution, potentially masks the details of the population, inducing miscalculations in ore reserves calculations.



(cont.) - Geochemical traverses through the ore. A: Fractal interpolation results; B: Linear interpolation results; C: complete traverse. Circles represent sample location. Vertical scaling in ppm. Horizontal axis representing distance from the theoretical origin.

Fractal interpolation algorithms, namely the Iterated Function Systems, exhibit a higher sensitivity for dealing with very complex populations such as geochemical distribution of elements. Traverses constructed from IFS routines have a complexity similar to the original population, reveal anomalies otherwise masked by linear interpolation routines, and forces the ratio anomaly/background to values closer to the unity.

These are promising results for the geochemical surveys specially in cases of poor sampling conditions.

The use of IFS for three-dimensional interpolation is a more complicated problem since the number of free parameters increases to four. The possibility of estimating the parameters from the data is presently under investigation.

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