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THE FRACTAL GEOMETRY OF AN ACTIVE FAULT (VILARIÇA STRIKE-SLIP FAULT, NE PORTUGAL) AND ITS IMPLICATIONS ON EARTHQUAKE GENERATION

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The fractal geometry of the Vilariça Fault was investigated. This fault is of strike-slip type, generated in Late-Variscan times and reactivated in Quaternary times with significant tectonic and seismic activity. Detailed mapping of the Fault north of the Douro River allowed the estimation of regional fractal dimension in three segments, from 1.0062 to 1.0171 and local fractal dimension of a set technique. It was found a negative correlation between fractal dimension and displacement; this mean that displacement increases as the fault plane is smoothed by fault propagation. This result suggests that fractal dimension could also control displacement for seismic event and so the magnitude of earthquakes generated along the active fault.

1. INTRODUCTION

Mandelbrot [1] introduced the concept of fractal, which is particularly convenient to describe the irregular geometry of many natural objects. For some of these objects, irregularity is observed from the microscopic to megascopic levels [2], as is the case with faults. We decided to test the idea of fractal dimension on a fault mapped in detail, and we choose a subvertical strike-slip fault, the Vilariça Fault (NE Portugal); it was selected in order to avoid the influence of topography, itself a fractal surface, on the geometry of the trace of the fault in map view.

Along active faults, earthquakes are generated by intrinsic irregularity of the geometry of the ruptured areas - the so

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called asperities and barriers - that extends on a wide range of scales and this could influence earthquake behaviour [3], [4]. This feature induced some authors to investigate the relationship between fractal geometry of active faults and earthquake behaviour on each fault [5], [6]. We decided to investigate the same relationship on the Vilariça Fault, because a lot of tectonic and seismotectonic data was available on this particular fault.

2. TECTONICS AND SEISMOTECTONICS OF THE VILARIÇA FAULT (NE PORTUGAL)

The Vilariça Fault [7] is a left lateral strike-slip fault that cut the Variscan basement in NE Portugal. It formed at the end of Variscan orogeny, around 300 MY ago, and extends in the NNE-SSW direction for 250 km between Puebla de Sanabria (Spain) and SW of Unhais da Serra (Portugal). The displacement along the fault is well established due to the abundant reference surfaces in the basement rocks (granitic bodies, axial traces of Variscan folds): it reaches a maximum of 8 km, in the Moncorvo central segment, and drops off smoothly towards the tips of the fault [8]; this allowed the construction of a slip profile along the fault, showing the variation of horizontal displacement along the trace of the subvertical strike-slip fault.

In Neogene and Quaternary times [9] the fault was reactivated, again as a strike-slip fault, and pull-apart basins, 2 to 3 km wide opened along it. Historical and instrumental seismicity is known along the fault but is more important in the Vilariça basin, which forms the central segment near Moncorvo; here the slip rate is estimated to be between 0.2 and 0.5 mm/year, during Quaternary times (last 2 MY).

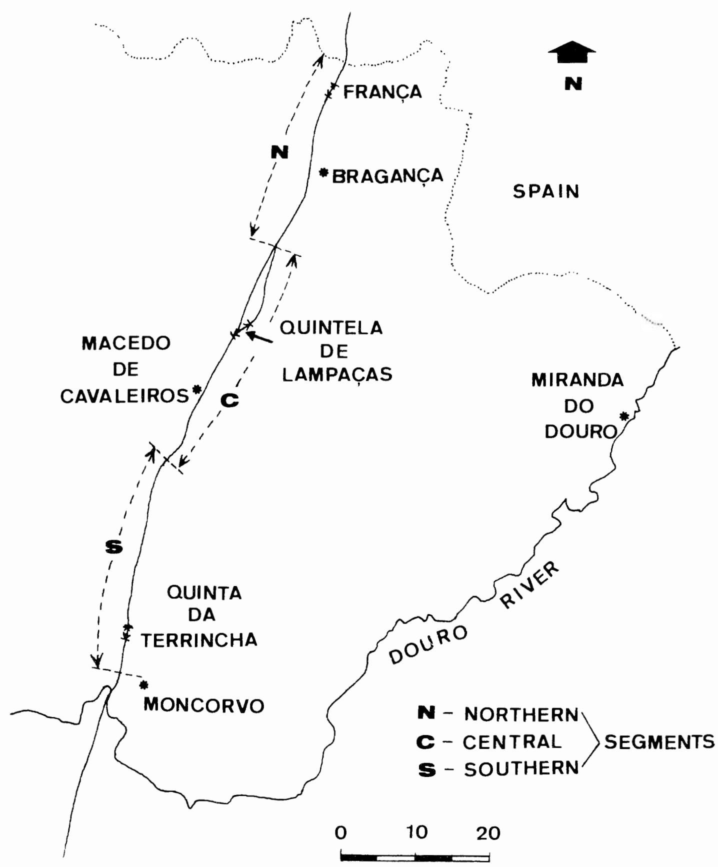
Fracturing and hydrothermal circulation along the Vilariça fault produced a variety of fault-rocks and quartz veins which are being subjected to detailed studies in some selected domains [10]. Detailed maps, at the 1/15.000 scale, were produced and were used to specify the fractal geometry at the mesoscale.

3.FRACTAL GEOMETRY OF THE VILARIÇA FAULT AT THE REGIONAL SCALE

To test the idea of fractal dimension of the Vilariça fault we used the empirical approach of Richardson [1]; we measured the fault length with different yardsticks at 3 available scales. The test was restricted to the domain situated at North of Moncorvo because no detailed mapping was available to the South of this area.

In the study area (North of Moncorvo), the fault was divided into three sectors, which were limited by significant variations of its trace geometry. Accordingly we considered a sector North of the duplex of Rebordaínhos, a central sector between Rebordaínhos and the bend of the fault trace near Bornes and a southern sector, south of this bend. (Fig. 1).

Fig. 1 - The Vilariça fault in Northeast Portugal



The length of each sector and total length were measured at various scales using the available geologic maps, namely 1/25.000, 1/200.000 and 1/1.000.000; a yardstick 5 mm long was used, which corresponds to 0.125 km, 1 km and 5 km, respectively.

With the results obtained we could construct a Richardson diagram, where we plotted the decimal logarithm, of the yardstick length versus the decimal logarithm, of total length, both in km; this was done for each sector and for the complete length of the fault. The slope of the best regression line among the 3 points on the plot for each sector and for the total length, is 1-D, where D is the fractal dimension (Table 1).

..Table I - Fractal dimensions of Vilariça Fault in Northern Portugal.

	Scale	1/25.000	1/200.000	1/1.000.000	Fractal
	Sector				Dimension
TOTAL LENGTH (km)	North	25.36	24.80	24.50	1.0094
	Centre	34.15	33.40	32.00	1.0171
	South	30.72	30.40	30.00	1.0062
	Total	90.23	88.60	86.50	1.0111

4. FRACTAL GEOMETRY OF THE VILARIÇA FAULT AT THE LOCAL SCALE

One of us [10] made a detailed map, at 1/15.000 scale of quartz fillings of multiple traces of Vilariça Fault at 3 localities North of Douro river. Those are Quinta da Terrincha, in the Southern sector, Quintela de Lampaças, in the Central sector and França, northern sector (Fig. 1).

The fractal dimension of complex multiple fault traces can be estimated using the covering dimension of a set technique [1], which was already applied to the San Andreas Fault [6].

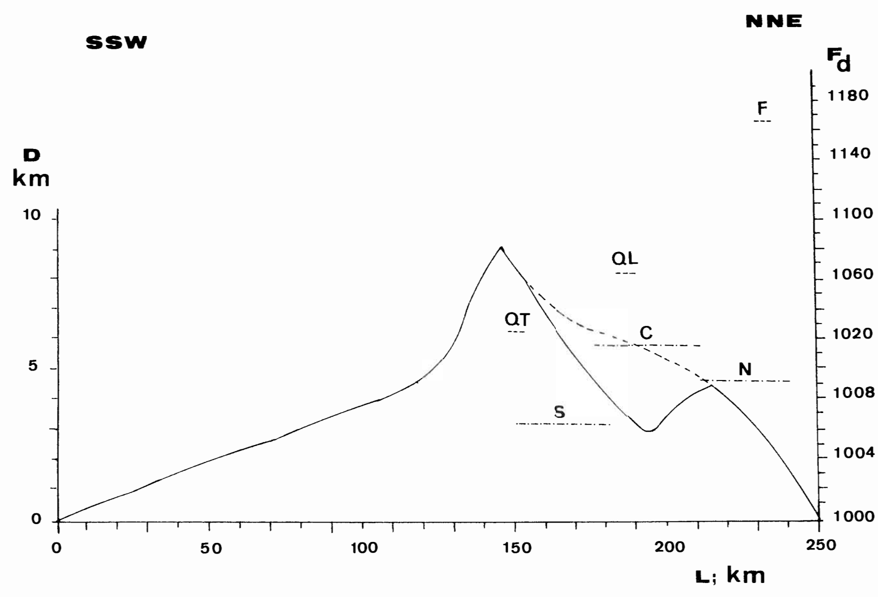
Each sector mapped in detail has a length of 2 - 3 km and a width around 0.75 km. Therefore the fractal dimension obtained is valid at a local scale.

The results obtained are as follows: Quinta da Terrincha has a fractal dimension of 1.0254; Quintela de Lampaças 1.0635 and França 1.1667.

5. RELATIONSHIP BETWEEN DISPLACEMENT AND FRACTAL DIMENSION IN THE VILARIÇA FAULT

Having obtained the fractal dimension of the Vilariça Fault at the regional and local scales, a logical step was to compare it with the observed displacement along the fault. To accomplish this we have plotted the observed regional and local fractal dimension values on the Vilariça Fault slip profile (Fig. 2). The presence of the strike-slip duplex [11] of Rebordaínhos should be taken in account, at the boundary between the central and northern sectors.

Fig. 2 - Displacement (**D**) vrs length (**L**) and fractal dimension (**Fd**) along the Vilariça fault. Vertical exageration 10x. Symbols of fault segments as in fig. 1; **QT**, **QL** and **F** for Quinta da Terrincha, Quintela de Lampaças and França sectors, respectively.



The regional fractal dimension was estimated along the main eastern trace, showed in the slip profile and the total slip was estimated summing the displacement of this trace and the smaller one along the western trace.

What can be deduced from the plot is that both the regional and local fractal dimensions increase from the positions at the central part of the fault towards its northern tip. There is a very clear negative correlation between displacement and fractal dimension; the physical significance of this correlation will be discussed below.

6. DISCUSSION OF TECTONIC AND SEISMOTECTONIC IMPLICATIONS

The roughness or bumpiness of natural surfaces are correctly described by Fractal Methods. They are a useful tool for describing the geometry of surfaces produced by fracture and should be easily correlated with concepts used in mechanics of fracture, such as surface energy. The behaviour of natural surfaces produced by fracture is controlled by their surface energy and this would be easily described by its fractal dimension. Indeed this has been recognised by application of fractal methods to faults and earthquake generation by tectonicists and seismologists, for example in the San Andreas Fault [5], [6].

If we compare the results obtained on the San Andreas fault against those obtained for the Vilariça Fault some regularities seem obvious. The fractal dimension measured on the main trace is lower (1.0008 to 1.0191 for S. Andreas; 1.004 to 1.011 for Vilariça) than the fractal dimension of the fault zone by the covering dimension of a set technique (1.1 to 1.4 for S. Andreas; 1.02540 to 1.16669 for Vilariça). The first parameter measures the irregularities of the main fault but the second one measures also the complexity of the entire fault zone. It is an open question to what domain of application each parameter would be more useful.

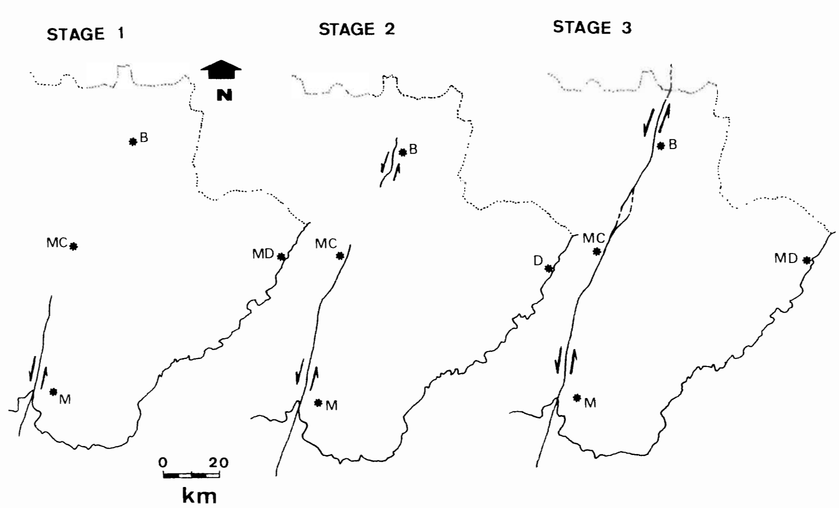
Comparing the results for S. Andreas and on Vilariça faults we conclude that the fractal dimension increases for the shorter fault, which is located in an intraplate tectonic environment. The conclusion seems logical since in longer, plate-boundary faults, renewed seismic activity will tend to smooth progressively the surface of a newly created active fault. Future work will show if this is or not a general rule, when more data on fractal dimensions of faults become available.

If fractal dimension is a controlling parameter of the mechanical behaviour of faults, as the studied cases tend to prove, we should consider it systematically on future studies.
For instance, a plot of logarithm of displacement versus logarithm of length [12] for strike-slip faults [8] show a large dispersion. However if we consider the brittle or ductile character of each fault we see that larger displacements are produced for more ductile faults than for more brittle faults. Could fractal dimension be a controlling parameter,

expressing the degree of ductility of each fault? If so, a fault with very low fractal dimension, as S. Andreas, would be characterised by very low shear stress acting on it [13], whereas a fault with higher fractal dimension, as Vilariça, would be characterised by higher shear stress [7], [9]. The implication of considering fractal dimension of faults would then be very deep, indeed.

The studies already made in S. Andreas and Vilariça show that fractal dimension varies along the fault. In Vilariça it has been shown [7] that the fault nucleated in the southern segment and propagated bilaterally to the northern and to the southern tips. As the southern segment is characterised by a lower fractal dimension at regional and local scales than the segments situated north of it variation in fractal dimension agrees with the tectonic evolution of the Vilariça fault. (Fig. 3).

Fig. 3 - Principal stages of proposed evolution for the Vilariça fault



The Vilariça fault nucleates in the domain around Moncorvo (southern segment) and propagates northward and southward. At a later stage, a second nucleation domain started its activity near Bragança in the Northern segment.

During an even later stage, the small misfit between the Moncorvo and Bragança fault traces generated the strike-slip duplex of Rebordaínhos; this occurred by propagation to the north on the northern tip of the Moncorvo segment and propagation to the south on the southern tip of the Bragança segment.

This evolution suggest a general rule for propagation of faults: these nucleate in weaker heterogeneities where highly irregular surfaces are generated. Then they propagate laterally and renewed stick-slip activity or stable sliding, smooth the oldest segments in contrast with rougher geometry of younger segments as one approaches the tips of the fault. This pattern of temporal and spatial evolution is clearly displayed in the case of Vilariça Fault.

The previous considerations can explain the observed reverse relationship between displacement and fractal dimension: displacement should increase as the fault becomes smooth, which is equivalent to a decrease in fractal dimension. If the behaviour of the fault is dominated by stick-slip versus stable-sliding, the increase in displacement should also be expressed as an increase of magnitude of earthquake events located in the fault, for the same rupture area. This is confirmed for the Vilariça Fault. In fact the more seismically active segment of the fault in Quaternary times is located in the Vilariça basin [9] where the fractal dimension of the fault trace has its lowest value; the detailed geometry of the active fault trace in Quaternary times seems to control the same general pattern of total displacement, both in Quaternary and Late-Variscan times; this suggest that the same general geometric trend was operative during both periods.

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Addendum

FRACTAL DIMENSIONAL ANALYSIS OF ACTIVE FAULTS: IMPLICATIONS ON MAXIMUM EARTHQUAKE MAGNITUDES

A. Ribeiro (D.G.F.C.L.)

The rupture length, l, of a maximum earthquake magnitude on active strike-slip faults of total length, L, and slip rate, S, obey an allometric law [14]:

$$l = a\ L^{b}\ S^{c} \qquad (1)$$

If the two faults have similar slip rates, the following expression is valid

$$l = a\ L^{b} \qquad (2)$$

Both l and L have fractal dimensions [5, 6] . Then in a double logarithimic Richardson plot [1], the following relationships must hold, if the two variables that are correlated have fractal dimensions:

$$\log l = \log K_l + \log\ (1 - D_l)\ \log \varepsilon \qquad (3)$$

$$\log L = \log K_L + \log\ (1 - D_L)\ \log \varepsilon \qquad (4)$$

We suppose that during a short increment of time, δt, fractal dimensions of rupture length and total length are maintained. This allow us to differentiate expressions 2 and 3 in relation to time, t:

$$d(\log l) = (1 - D_l)\ d(\log t) \qquad (5)$$

$$d(\log L) = (1 - D_L)\ d(\log t) \qquad (6)$$

In the allometric law, b is the relative growth rate

$$b = \frac{\dfrac{dl}{dt} \cdot \dfrac{1}{l}}{\dfrac{dL}{dt} \cdot \dfrac{1}{L}} \qquad (7)$$

and can be calculated by dividing expression 4 by expression 5:

$$b = (1 - D_l)\ (1 - D_L)^{-1} \qquad (8)$$

So, in the present case

$$l = l_o \, L^{[(1-D_l)/(1-D_L)]} \qquad (9)$$

and D_L is the fractal dimension of the total fault length and D_l is the fractal dimension of the rupture length, during the maximum magnitude event. Both D_l and D_L must have dimensions of irregular curves, between 1 and 2.

The fractal dimension of a fault trace rupturing in a single event of maximum magnitude must be less than the fractal dimension of the total fault trace; in fact, if the fault trace increases in irregularity the rupture tend to stop its propagation. So, maximum earthquake magnitude should occur if

$$2 > D_L > D_l > 1 \qquad (10)$$

An immediate consequence of expression (10) is

$$(1 - D_l)\,(1 - D_L)^{-1} < 1 \qquad (11)$$

and the existing data show that the parameter b, relative growth ratio, is in fact less than 1 [14].

This confirmes our previous view [14] that allometry imposes that rupture length must grow slower than the total length, in contrast with opposite conclusions by Slemmons [15] . Fractal dimensional analysis of active faults show that shorter faults are proportionally more hazardous than longer ones, with obvious implications on seismic risk studies. The expression (9) provides the theoretical basis for the segmentation of active faults by studing the variability of fractal dimension along each active fault trace. This multifractal approach allows us to assign a specific rupture length for each segment and from that rupture length to estimate the maximum earthquake magnitude.

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