Comun. Serv. Geol. Portugal, 1991, t. 77, pp. 53-58

Comunicações dos
Serviços Geológicos
de Portugal

A1

A fractal approach to tectonic deformation

A. RIBEIRO*, A. MATEUS* & R. DIAS*

Key-words: Tectonic deformation; (multi)fractals; anisotropic fractoids; allometry in tectonics.

Abstract: Tectonic deformation occurs on a large range of scales in space and time. This stimulates the application of fractal geometry concepts to tectonic deformation in discontinuous and continuous media. An extension of the original concept of fractal has already been proposed to include anisotropic fractal objects. Discontinuous deformation in the brittle regime can be characterised by multifractals. As a consequence, fault surfaces should be anisotropic fractoids and their dimension of tensorial nature.

Continuous (plastic) yielding mechanisms are indeed closely interconnected with discontinuous processes, and thus a fractal approach to the former is also desirable. In fact the propagation of many plastic structures is geometrically controlled by non Euclidian intracrystalline microfractures or dislocation arrays, which have anisotropic allometric growth explained by fractal dimensional analysis.

Resumo: A deformação tectónica ocorre a todas as escalas de tempo e espaço. Tal facto sugere a aplicação dos conceitos de geometria fractal à deformação natural das rochas em meio descontínuo e contínuo. A extensão do conceito original de fractal já foi proposta de modo a incluir objectos fractais anisótropos. A deformação descontínua típica do regime frágil pode ser caracterizada em termos de multifractais. Como consequência, as superfícies de falha deverão constituir exemplos de fractóides anisótropos, cuja dimensão deverá ser de natureza tensorial.

É possível estabelecer relações de dependência entre os mecanismos de cedência contínua (plástica) e os processos responsáveis pela deformação descontínua das rochas, o que torna desejável uma abordagem fractal aos primeiros. De facto a propagação de muitas estruturas plásticas é geometricamente controlada por microfracturas intracristalinas ou arranjos de deslocações com geometria não Euclidiana e cujo desenvolvimento anisótropo e alométrico é explicado pela análise de dimensão fractal.

A Tectónica e Sismotectónica constituirão, sem dúvida alguma, um campo vasto e promissor para a aplicação da teoria de sistemas dinâmicos.

INTRODUCTION

The deformation of rock materials can be approached in a variety of ways. The theoretical physicist will be interested in deformation as a symmetry-breaking process obeying laws of statistical mechanics. The rock-mechanicist will be interested on a experimental approach, monitoring deformation in the laboratory, as well as in the field. The structural geologist will be mainly interested on natural yielding mechanisms and use experimentation only as a tool to explain them. However, irrespectively of the purpose, tectonic deformation occurs on an enormous range of scales (from 10^3 km to 10^{-10} m) and strain rates (from 10^{-18} s^{-1} to 10^2 s^{-1}); so, can we use the fractal approach as a useful tool in the unification of such range of space and time scales? As a matter of fact, many laws in tectonics, and in rheological behaviour, relating stresses and strains can be estabilished as power-law functions, which have expression in log-log graphs. Thus fractal dimensional analysis can be an explanation for empirical data and laws where dimensions (constant empirical values) are often different from integers. Note that fractals are only one branch of the immense field of *Dynamic Systems,* which will be of extreme utility in the study of complex processes in the Earth Sciences (e.g. RIBEIRO, in press).

In the first part of the paper we will discuss the extension of the fractal concept to include anisotropic objects, such as those that appear more fre-

---

* Departamento de Geologia da Faculdade de Ciências da Universidade de Lisboa, Ed. C2, Piso 5, Campo Grande — 1700 Lisboa.

quently in tectonics; we intend to explore the example of *fault surfaces as anisotropic fractoids.* Discontinuous deformation by fracture in the brittle regime will be more amenable to the fractal approach (RIBEIRO *et al.,* 1990) than continuous deformation. Nevertheless, we will show in the second part of the paper that tectonic processes change from continuous to discontinuous as the scales change, and the fractal analysis can be used even in objects that experienced a bulk plastic deformation.

ANISOTROPIC FRACTOIDS: THE EXAMPLE OF FAULT SURFACES

The concept of fractal (MANDELBROT, 1982) has been proved to be very useful in the characterisation of natural objects such as coast-lines, clouds, topographic surfaces of Earth and other Planets (for a review see KAYE, 1989), and tectonic fault surfaces (AVILES *et al.,* 1987; OKUBO *et al.,* 1987; RIBEIRO *et al.,* 1990). In fact no true fractals exist in nature because there is an «inevitable break down (of fractal dimension) well before the molecular scale is reached» (FALCONER, 1990, p. 265). But the fractal nature of natural objects is a good approximation for a specific large range of scales; for instance for fault surfaces from 0.5 to 1000 km (AVILES *et al.,* 1987).

As the domain of applicability of fractal geometry was widening, the initial concept of fractal, based on the notion of strict self-similarity, was extended to include cases of self-affinity (MANDELBROT, 1982), and multifractals (for a review see FALCONER, 1990) where fractal dimension is not constant and varies as a function of scale.

It was shown for studied fault surfaces that these were in fact multifractals (AVILES *et al.,* 1987; OKUBO *et al.,* 1987). A further generalisation is obvious in the case of fault surfaces: the fractal dimension along a fault trace in some planar surface (that can be estimated by a great variety of methods, see FALCONER, 1990), should be different for different fault trace directions. It is a fact of experience for any tectonicist that the fault surface irregularity is different in the direction of movement or in a perpendicular direction to it (SCHOLZ, 1990, p. 147). Or in terms of kinematic axes, the fault trace in the *ac* plane of movement should have a different fractal dimension from a fault trace in the *ab* plane; and these axes are materialised in the fault trace by a variety of features (for a review see, e g., RAMSAY & HUBER, 1987).

In the case of *strict* and *statistical self-similarity* and *self-affinity,* the fractal dimension should not be different with direction by definition; a surface can be characterised by a single number independent of direction or, in other words, *fractal dimension is of scalar nature.* The fractal dimension of the surface, $D_{surface}$, can be calculated from the fault dimension of any trace, D_{trace}, using the relationship

$$D_{surface} = D_{trace} + 1$$

that has been proved for fracture surfaces of metals (MANDELBROT *el al.,* 1984). The theoretical basis for this is simply the consideration of topological and fractal dimension concepts (MANDELBROT, 1982). If an anisotropy is present, as in a wide range of usual and natural objects, the strict concept of self-similarity is not observed. But we can extend the concept of fractal and speak of «*anisotropic fractoids*» with «*multiple elliptical dimensions*» (LOVEJOY *et al.,* 1987). An example is the geometry of clouds (and rain) due to atmosperic stratification; stratocumulus are obviously anisotropic! *In anisotropic fractoids fractal dimension is of tensor nature.* Fault surfaces should be anisotropic fractoids and the fractal dimension of a fault surface cannot be deduced from a single fault trace, as in the case of the Hausdorff-Besicovich dimension (AVILES *et al.,* 1987; OKUBO *et al.,* 1987).

A simple symmetry argument can guide our thinking and our research. If the generating process is highly symmetric the fractal geometry of the product would reflect it. For instance an explosion, or the Big-Bang, would produce an isotropic, although inhomogeneous, distribution of fragments and matter-radiation, respectively, with spherical symmetry. A tensile axial test on a cubic metal would produce a fracture surface with axial symmetry; pressure solution with axial symmetry would also produce stylotites with fractal dimension much higher than a fault surface (between 2 and 3) but independent on the magnitude of maximum compressive stress, irrespective of intermediate and minimum stresses. In a fault or shear zone the symmetry of the generating process is orthorrombic or lower, and the geometry of the fault surface should be anisotropic; and similar, for a seismogenic process along an active fault with a double--couple generating mechanism.

These ideas can, and will be tested by the contouring of a natural, well exposed, and as large as possible fault surface at the scale of 10^1 to 10^{-2} m and by computer simulations.

CONTINUOUS AND DISCONTINUOUS DEFORMATION

The behaviour of rocks during tectonic processes, namely polymineralic, shows that deformation can be discontinuous at submicroscopic-microscopic scales (heterogeneous intracrystalline slip induced by dislocation movement and/or intragranular cracking) and continuous at mesoscopic levels (bulk plastic deformation, expressed in the language of Continuum Mechanics); again discontinuous at higher levels (mesoscopic plastic yielding related with the development of narrow macroscopic shear zones) and finally continuous at even higher level (megascopic continuous deformation by distributed brittle faulting). A great diversity of methods has been developed to quantify continuous deformation at some higher levels from discontinuous deformation at a lower level (e.g. MOLNAR, 1983; RECHES, 1983). However, the spatial coexistence and the interdependence of active plastic and brittle mechanisms in the lithospheric crust, suggest that a fractal approach can be useful for the connection and subsequent characterisation of strain at the full range of scales. In this respect we will refer to objects that are not fractals at the same scale but controlled by fractal entities and (deterministic?) behaviour at a lower scale, using the image of **allometry** in tectonics (RIBEIRO, 1990; RANNALI, 1980).

A typical example of allometry is the relative development of the human beings head when compared to other parts of the body (legs, trunk, total height), during their growth. None of the length dimension of the chosen variables in fractal, but the growth of each part is essentially controlled by the development of the fibro-cartilaginous and muscles masses, and blood and nervous networks, which are typically fractal entities. Thus we can say that these variables are «fractally controlled». A tectonic analogue of the allometric relationship should be the generation of the strain elliptical marker non-fractal obtained from a circular marker by the development of plastic and/or brittle yielding microstructures with a fractal distribution. As a matter of fact, under appropriate P-T conditions the progression of slip systems at the intracrystalline level is constrained by the cooperative behaviour of different dislocation sets. Their propagation enable the creation of new point defects and other «debris» necessary for the multiplication of non Euclidian linear defects, or it may result in low-energy arrays or irregular tridimensional networks of dislocations that are fairly stable with respect to glide and climb. (e.g. NICOLAS & POIRIER, 1976; KERRICH & ALLISON, 1978; WHITE, 1976; BARBER, 1985).

During rock deformation, recovery mechanisms compete effectively with work-hardening processes (e.g. POIRIER & GUILLOPÉ, 1979; LANGDON, 1985). The former comprises the climb-assisted annihilation of dislocations of opposite sign, the achievement of equilibrium concentrations of point defects, and dislocation climb to form sub-boundaries often with irregular geometry; subsequent dynamic recrystallisation lead the nucleation of «new» intra and/or intergranular grains, developed following power laws of growth (e.g. WHITE, 1977; GOTTSTEIN & MECKING, 1985). Work-hardening embraces the generation of dislocations on glide planes (essentially constrained by Schmid's law, defect chemistry of the minerals, and P-T conditions), and subsequent creation of inhomogeneous distribution of the above linear anisotropies, with tight forming in some regions (e.g. GOTTSTEIN & MECKING, 1985; VAN HOUTTE & WAGNER, 1985).

At this point it is interesting to point out that the principal mechanism of dislocation generation, the Frank-Read source, produces a *similar succession of dislocation loops* in the glide plane. Under different P-T conditions and a specific geochemical environment, syntectonic dislocations can be also generated at the tips of intragranular cracks, at the intersections of deformation bands, by twins and their interaction with grain boundaries, and at second phase particles (e.g. ATKINSON, 1984; MacDONALD, 1987; NEUMANN, 1987; McMEEKING, 1987). These geometric variables show, at least, a fractal distribution and are responsible for different degrees of linear work-hardening at low temperature conditions.

Grain-boundary mechanisms, including intergranular slip at temperatures that anable the activation of diffusion creep processes, and grain-boundary hardening and/or pressure solution, coupled with intergranular stress-corrosion responsible for sub-critical propagation of microfractures at medium-lower temperatures, are also extremely important in continuous deformation (e.g. Kerrich, 1977; Kranz, 1983; Atkinson, 1987; De Hosson & Vitek, 1987; Ingraffea, 1987; Losch, 1987; Knipe, 1989). Their dependence on the geometry of the grain boundaries is obvious, and their development are certainly constrained by the fractal dimension of the former.

Allometry can also explain the non linear relationship between length and displacement for strike-slip faults (Rannali, 1980) and many empirical relationships in Seismotectonics (Ribeiro *et al.,* 1990); allometry is then simply the consequence of maintaining, during a short interval of time, a constant fractal dimension for the variables (geometric entities) involved in rock deformation. So, using the fractal dimensional analysis (Ribeiro *et al.,* 1990), the allometric relationship between two variables, I and L

$$I = a\ L^{b}$$

can be expressed by

$$I = 1_0^{[(1-DI)/(1-DL)]}$$

where D_1 and D_L are the fractal dimension of the variables, and a, b and 1_0 constants.

A logarithmic strain plot (Ramsay & Huber, 1983) is often used by tectonicists in order to express domains in finite strain and strain paths in progressive deformation. In some cases, strain paths are straight lines, and these correspond to allometry with b, relative growth ratio, constant; otherwise, more complex curves will result. How do we relate constant strain path with allometry? If some deformation mechanism is operating (plastic flow, pressure solution, cataclastic flow, etc.) under an anisotropic stress state, the multiple elliptic dimension parallel to the maximum compressive stress and parallel to the minimum compressive stress will be different; nevertheless, their ratio can remain constant with time, and allometry will inevitably result. If the ratio changes, a more complex strain path will result, and as the dimension is fractal the whole log strain plot will be occupied.

Fractal dimensional analysis can also explain the exponents from 3 to 8 in non linear power laws for creep. In fact, if some mechanism depends on the length dimension with integer exponents of 1, 2 and 3 in linear phenomena, and if we consider the non-linear counterparts with $1 < L^n < 3$, fractal exponents with values between 1 and 9 can result.

CONCLUSION

Given the very large range of scales in tectonic deformation a multifractal approach to tectonic deformation will be certainly more useful than a simplistic unidimensional approach independent of scale observations. There are in fact similarities between plastic-brittle yielding at the atomic and microscopic scales, and macroscopic rupture related with seismic mechanisms. These similarities have been used to calculate stress and strain fields independently of scale using the Elasticity Theory. So each scale will have to use both the unified fractal approach and a specific treatment («*scalar methodology*» of Glangeaud, 1960).

ACKNOWLEDGMENTS

The authors wish to express their appreciation to Fernando J. A. S. Barriga for insightful comments on an earlier version of the manuscript.

REFERENCES

Atkinson, B. K. (1984) – Subcritical crack growth in geological materials. *J. Geophys. Res.,* **89 (B6)**: 4077-4117.

Aviles, C. A.; Scholz. C. H.; Boatwright, J. (1987) – Fractal analysis applied to characteristics segments of the San Andreas Fault. *J. Geophys. Res.,* **92 (B1)**: 331-344.

Barber, D. J. (1985) – Dislocations and microstructures. *In*: «*Prefered orientation in deformed metals and rocks: an introduction to modern texture analysis,* Ed. by Wenk H. R., Academic Press Inc., London: 149-182.

De Hosson, J. Th. M.; Vitek, V. (1987) – Structure of grain boundaries and interfaces. *In*: «*Chemistry and Physics of Fracture*», Ed. By Latanision R. M. & Jones R. H., NATO ASI Series, Serie E: Applied Sciences – N° 130, D. Reidel Publ. Co., Boston: 63-84.

Falconer. K. (1990) – Fractal Geometry-Mathematical Foundations and Application. J. Wiley & Sons, New York, **288** pp.

Glangeaud, L. (1960) — La part de la géologie dans les grandes synthèses; comment la géodynamique explique l'évolution de la Terre. *Encyclopédie Francaise,* Introduction à la Geólogie, Larousse, Paris: 13-24.

Gottstein, G.; Mecking, H. (1985) — Recrystallization. *In*: «*Preferred orientation in deformed metals and rocks: an introduction to modern texture analysis,* Ed. by Wenk H. R., Academic Press Inc., London: 183-218.

Ingraffea, A. R. (1987) — Theory of crack initiation and propagation in rock. *In*: «*Fracture mechanics of rock*», Ed. by Atkinson, B. K., Academic Press Inc., Geology Series, London: 71-110.

Kaye, B. H. (1989) — A random walk through fractal dimensions. VCH, Weinheim, **421** pp.

Kerrich, R. (1977) — An historical review and synthesis of research on pressure solution. *Zbl. Geol. Paleont.,* **I(H-5/6)**: 512-550.

Kerrich, R.; Allison, I. (1978) — Flow mechanisms in rocks. *Geoscience Canada,* **5(3)**: 109-118.

Knipe, R. (1989) – Deformation mechanisms – recognition from natural tectonites. *J. Struct. Geol.,* **11(1/2)**: 127-146.

Kranz, R. L. (1983) — Microcrack in rocks: a review. *Tectonophysics,* **100**: 449-480.

Langdon, T. G. (1985) — Regimes of plastic deformation. *In*: «*Preferred orientation in deformed metals and rocks: an introduction to modern texture analysis,* Ed. by Wenk H. R., Academic Press Inc., London: 219-232.

Losch, W. (1987) — Intergranular fracture. *In*: «*Chemistry and Physics of Fracture*», Ed. by Latanision R. M. & Jones R. H., NATO ASI Series, Serie E: Applied Sciences — N° 130, D. Reidel Publ. Co., Boston: 461-491.

Lovejoy, S.; Schertzer, D.; Tsonis, A. A. (1987) — Functional box-counting and multiple elliptical dimensions in rain. *Science,* **235**: 1036-1038.

MacDonald, D. D. (1987) — Surface chemistry in aqueous solutions. *In*: «*Chemistry and Physics of Fracture*», Ed. by Latanision R. M. & Jones R. H., NATO ASI Series, Serie E: Applied Sciences — N° 130, D. Reidel Publ. Co., Boston: 255-286.

Mandelbrot, B. B. (1982) — The fractal geometry of nature. W. H. Freeman & Company Ed., New York, **468** pp.

Mandelbrot, B. B.; Passoja, D. E.; Paullay, A. J. (1984) — Fractal character of fracture surfaces of metals. *Nature,* **308**: 731-722.

McMeeking, R. M. (1987) — Plastic flow instabilities. *In*: «*Chemistry and Physics of Fracture*», Ed. by Latanision R. M. & Jones R. H., NATO ASI Series, Serie E: Applied Sciences — N° 130, D. Reidel Publ. Co., Boston: 58-128.

Molnar, P. (1983) — Average regional strain due to slip on numerous faults of different orientations. *J. Geophys. Res.,* **88**: 6430-6432.

Neumann, P. (1987) — Plastic processes at crack tips. In: «*Chemistry and Physics of Fracture*», Ed. by Latanision R. M. & Jones R. H., NATO ASI Series, Serie E: Applied Sciences — N° 130, D. Reidel Publ. Co., Boston: 63-84.

Nicolas, A.; Poirier, J. (1976) — Crystalline plasticity and solid state flow in metamorphic rocks. John Wiley & Sons, New York, **444** pp.

Okubo, P. G.; Aki, K. (1987) — Fractal geometry in the San Andreas Fault Syistem. *J. Geophys. Res.,* **92(B1)**: 331-344.

Poirier, J. P.; Guillopé, M. (1979) — Deformation induced recrystallization of minerals. *Bull. Minéral.,* **102**: 67-74.

Ramsay, J.; Huber, M. (1983) — The technique of modern structural geology. Vol. I: Strain analysis. Academic Press Inc., London, **307** pp.

—— (1987) — The technique of modern structural geology. Vol. II: Folds and fractures. Academic Press Inc., London, **397** pp.

Rannali, G. (1980) — A stochastic model for strike-slip faulting. *Mathem. Geol.,* **12**: 339-412.

Reches, Z. (1983) — Faulting of rocks in three-dimensional strain fields. II — Theoretical analysis. *Tectonophysics,* **95**: 133-156.

Ribeiro, A. (in press) — Order and chaos in the magnitude of earthquake faulting. Submited to *Acad. Ci. Lisboa,* Portugal.

Ribeiro, A.; Mateus, A.; Moreira, M. C.; Coutinho, M. F. (1990) — The fractal geometry of an active fault (Vilariça, strike-slip fault, NE Portugal) and its implications on earthquake generation. *Proc. of FRACTAL 90,* 1st, **IFIP Conf. on Fractals,** Fund. Calouste Gulbenkian, Lisboa, Portugal.

Scholz, C. H. (1990) — The mechanisms of earthquakes and faulting. Cambridge Univ. Press, Cambridge, **439** pp.

Van Houtte, P.; Wagner, M. (1985) — Development of textures by slip and twining. *In*: «*Preferred orientation in deformed metals and rocks: an introduction to modern texture analysis,* Ed. by Wenk H. R., Academic Press Inc., London: 233-259.

White, S. H. (1976) — The role of dislocation processes during deformation, with particular reference to quartz. *In*: «*The physics and chemistry of minerals and rocks*», Ed. by Strens, R. G. J., John Wiley & Sons Inc., New York: 75-91.

—— (1977) — Geological significance of recovery and recrystallization process in quartz. *Tectonophysics,* **39**: 143-170.