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GEOID DETERMINATION BY GRAVITY SPACE APPROACH

DISSERTATION THESIS IN PHYSICAL GEODESY

Carlos Antunes

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Carlos ANTUNES
cmantunes@fc.ul.pt
Departamento de Matemática da Faculdade de Ciências da Universidade de Lisboa
Rua Ernesto Vasconcelos, Edifício C6, 1974-016 LISBOA, Portugal

ABSTRACT

The gravity space approach transforms the free boundary-value problem into a fixed boundary-value problem, giving a great mathematical contribution to the boundary problem of physical geodesy. This investigation is conducted on the applicability of the approach to the determination of geoid models through its linear formulation. In this approach, the combination of two different data sets, gravity and direction of the vertical (equivalent to gravity anomaly and deflection of the vertical on the conventional theory) through the adjoint potential defined in the gravity space enables the determination of a combined geoid solution, which is only possible in conventional terms with the least squares collocation treatment. Due to the fact that the solution of this approach is a recursive function, the geoid determination is only possible by an iterative procedure, which requires an *a priori* geoid solution. Therefore, the presented method is considered as an improving method rather than a determination method. Numerically, the solution is obtained by the application of a point mass method in the space domain, using a harmonic functional as an approximation to the problem's solution.

This new approach, presented in 1978 by *Fernando Sansó*, became a theory to solve the uniqueness of the Molodensky problem reducing it into a Dirichlet problem in an auxiliary space, called the gravity space. Based on this approach the free boundary-value problem of physical geodesy is transformed into a fixed boundary-value problem, where the existence and the uniqueness of the solution are guaranteed. This treatment of the boundary problem by *Sansó* provides a very original and much simpler new approach. However, on the practical side no work is known until the present date.

The presented work is an attempt to improve new solutions, exploiting the new approach of the boundary-value problem, with the purpose to experience the possibility to obtain a better precision on the geoid models when compared with the usual approaches that have been used for the last several decades.

A great disadvantage that can be pointed out to the applicability of the gravity space approach is related to the characteristic of the necessary data to be applied, the state and gravity vectors at every point of observation. Just recently it was possible to observe, at any point on the earth surface, all the observations needed to compute those quantities with the required precision and time consuming. Since astronomic and geodetic positions and gravity observations are involved, the application of this approach has been practically impossible because it has been very difficult to do all of the observations in a considerable short period of time, at least, up to 10 or 15 years ago. Another disadvantage associated to this characteristic is the fact that the astronomic coordinates can only be observed with the desirable precision on land, it is not possible to do it on the sea. This would turn the approach only appropriate to continental regions of the earth surface, remaining the problem for global models determination unsolved. In fact, this is not quite a problem; it can be easily overtaken, because the deflections of the vertical can be computed from the shipboard observed gravity anomalies and from inversion of the satellite altimetry observations.

The great demand on the data composition, as well as the problem of its application in the ocean areas, led us to formulate a combined linear formulation in the gravity space, to enable the use of the actual heterogeneous data (gravity anomalies and deflections of the vertical on different point distribution). The gravity vector at every data point is just not possible with the usual data distribution. Through the combination of two boundary conditions, imposed on the two different data points (gravity and deflections of the vertical points), one can establish the determination of the anomalous adjoint potential in gravity space at every data point. In other words, for this combined solution, we have combined the vector treatment with scalar treatment of the boundary-value problem in gravity space approach. Based on the anomalous potential boundary values, the geoid determination is performed as in the simple formulation. The numerical results of this approach are compared to the solutions obtained by conventional methods.

On the non-linear formulation of the boundary-value problem in gravity space approach, a physical model is introduced and a numerical method based on superposition of gravity sources is designed (gravity confinement method). Unfortunately, due to the fact that the respective differential equation, which is non-linear, does not admit the superposition with the solution found. Final results were not obtained and consequently it was not possible to conclude about its applicability.

The linear formulation solution is successfully applied to gravity, astronomy and GPS observations collected in the Tagus Valley region during the PhD program. Concerning the astronomic observations of latitude and longitude, a real time digital system (ICARUS) was successfully applied to reduce the time effort maintaining the necessary precision.

This work consists on an initial analysis of the gravity space approach for the geoid determination. Considering its simplicity for solving the boundary-value problem, it brings strong possibilities for solving the problem even in the non-linear formulation. The limitations of this application in a local domain did not disable the possibility to propose it as an alternative for the known conventional geoid determination methods.

We also assume that this investigation is also a possibility to propose some guidelines for the research in physical geodesy concerning the theoretical boundary-value problem, and new ideas for subjects to be studied on the practical computation and research in the precise geoid determination.

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