Faculdade de Ciências da
Universidade de Lisboa
Departamento de Matemática

Matemática II
2º Semestre de 2009/2010

Exercício - teste 8

a) Calcular $\int \cos^3 x \, dx$.

Resolução:

$$\int \cos^3 x \, dx = \int \cos^2 x \, \cos x \, dx = \int (1 - \operatorname{sen}^2 x) \cos x \, dx$$

$$= \int \cos x \, dx - \int \operatorname{sen}^2 x \, \cos x \, dx = \operatorname{sen} x - \frac{\operatorname{sen}^3 x}{3} + C \ .$$

b) Calcular $\int x^2 \operatorname{sen} 5x \, dx$.

Resolução:

$$u = x^2, \qquad dv = \operatorname{sen} 5x \, dx,$$

$$du = 2x \, dx, \qquad v = \int \operatorname{sen} 5x \, dx = -\frac{\cos 5x}{5},$$

Logo,

$$\int x^2 \operatorname{sen} 5x \, dx = -x^2 \frac{\cos 5x}{5} + \frac{2}{5} \int x \cos 5x \, dx$$

$$u = x, \qquad dv = \cos 5x \, dx,$$

$$du = dx, \qquad v = \int \cos 5x \, dx = \frac{\operatorname{sen} 5x}{5},$$

Logo,

$$\int x^2 \operatorname{sen} 5x \, dx = -x^2 \frac{\cos 5x}{5} + \frac{2}{5} \left(\frac{1}{5} x \operatorname{sen} 5x - \frac{1}{5} \int \operatorname{sen} 5x \, dx \right)$$

$$= -x^2 \frac{\cos 5x}{5} + \frac{2}{25} x \operatorname{sen} 5x + \frac{2}{125} \cos 5x + C \ .$$

BÓNUS. Calcular $\int e^{3x} \cos 5x \, dx$.

$$u = e^{3x}, \qquad dv = \cos 5x \, dx,$$

$$du = 3e^{3x} \, dx, \qquad v = \int \cos 5x \, dx = \frac{\operatorname{sen} 5x}{5},$$

Logo,

$$\mathrm{I} = \int e^{3x}\cos 5x\ dx = \frac{1}{5}e^{3x}\,\mathrm{sen}\ 5x - \frac{3}{5}\underbrace{\int e^{3x}\,\mathrm{sen}\ 5x\ dx}_{\mathrm{II}}$$

$$\boxed{\begin{array}{ll} u = e^{3x}, & dv = \mathrm{sen}\ 5x\,dx, \\ du = 3e^{3x}\,dx, & v = \displaystyle\int \mathrm{sen}\ 5x\ dx = -\frac{\cos 5x}{5}, \end{array}}$$

onde

$$\mathrm{II} = \int e^{3x}\,\mathrm{sen}\ 5x\ dx = -\frac{1}{5}e^{3x}\cos 5x + \frac{3}{5}\underbrace{\int e^{3x}\cos 5x\ dx}_{\mathrm{I}}$$

e portanto

$$\begin{aligned} \mathrm{I} &= \frac{1}{5}e^{3x}\,\mathrm{sen}\ 5x - \frac{3}{5}\left(-\frac{1}{5}e^{3x}\cos 5x + \frac{3}{5}\mathrm{I}\right) \\ &= \frac{1}{5}e^{3x}\left(\mathrm{sen}\ 5x + \frac{3}{5}\cos 5x\right) - \frac{9}{25}\mathrm{I} \\ &= \frac{\frac{1}{5}e^{3x}\left(\mathrm{sen}\ 5x + \frac{3}{5}\cos 5x\right)}{\frac{34}{25}} + C \end{aligned}$$

Em resumo,

$$\int e^{3x}\cos 5x\ dx = \frac{5}{34}\,e^{3x}\left(\mathrm{sen}\ 5x + \frac{3}{5}\cos 5x\right) + C\ .$$