Faculdade de Ciências da
Universidade de Lisboa
Departamento de Matemática

Miscelânea

Gráficas de algumas funções

Ideia principal: A partir da gráfica de algumas funções modelo esboçamos a gráfica de funções similares usando translações, mudança de escalas nas variáveis e reflexões nos eixos coordenados, bem como empregrando o valor absoluto para reflectir a parte da gráfica que estiver debaixo do eixo horizontal. Note-se que nem sempre pode-se realizar estas operações todas, o domínio das funções modelo é importante.

Exemplo 1

$$\begin{array}{rcl} \text{Dom } f & = & \mathbb{R} \\ \text{Im } f & = & [0, +\infty[\end{array}$$

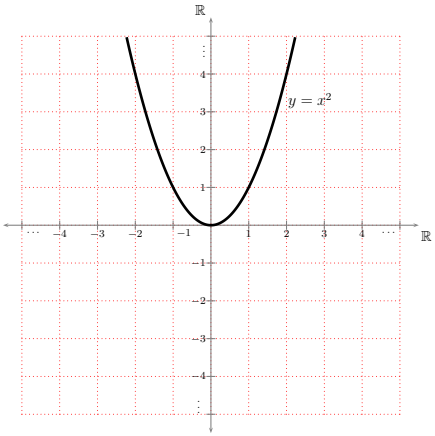

Figura 1: Modelo: $y = f(x) = x^2$

Reflexão no eixo vertical

$x \to -x$

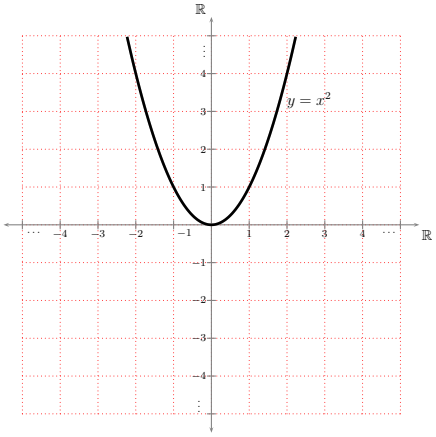

Figura 2: $y = f(-x) = (-x)^2 = x^2$

Reflexão no eixo horizontal

$y \to -y$

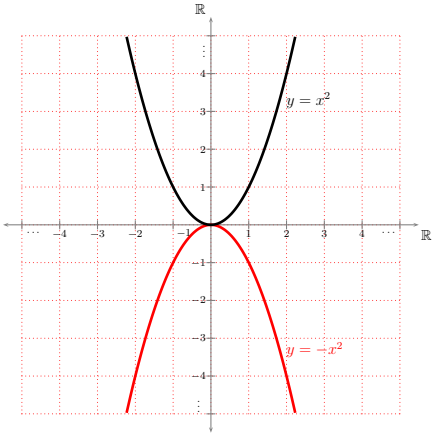

Figura 3: $y = -f(x) = -x^2$

Mudando a escala de x

$$x \to ax \quad , \quad a \in \mathbb{R}$$

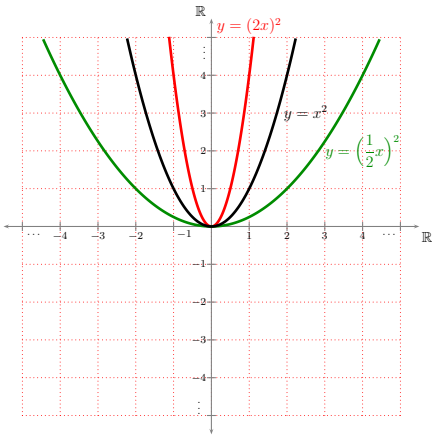

Figura 4: $y = f(ax) = a^2x^2$

Mudando a escala de y

$$y \to cy \quad , \quad c \in \mathbb{R}$$

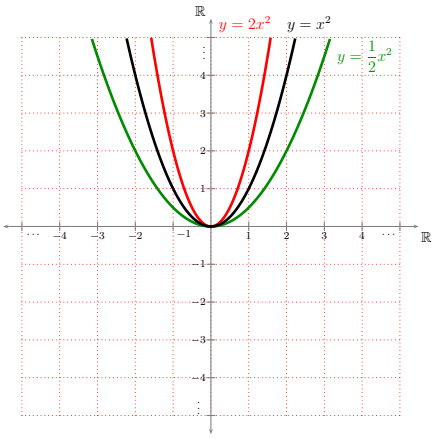

Figura 5: $y = cf(x) = cx^2$

Translação horizontal

$$x \to x + b \quad , \quad b \in \mathbb{R}$$

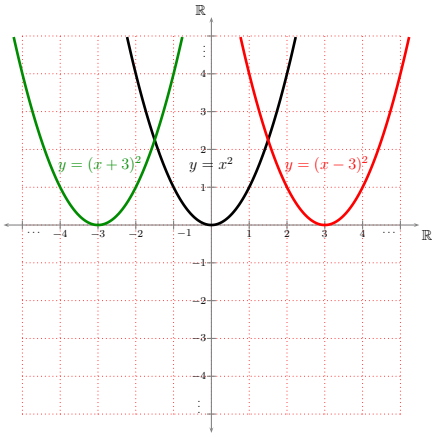

Figura 6: $y = f(x + b) = (x + b)^2$

Translação vertical

$$y \to y + d \quad , \quad d \in \mathbb{R}$$

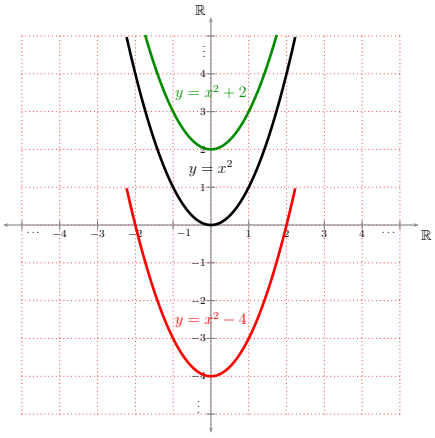

Figura 7: $y = f(x) + d = x^2 + d$

Todas juntas

$x \mapsto ax + b \ , \ y \mapsto cy + d$

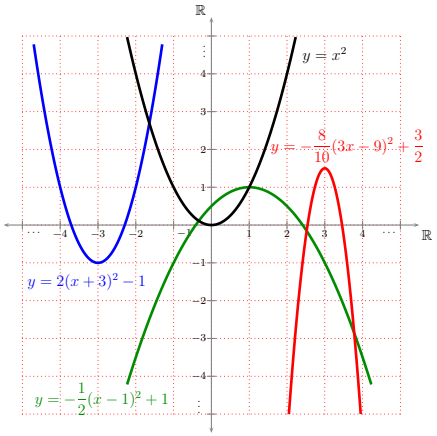

Figura 8: $y = c(ax + b)^2 + d$

Valor absoluto

$x \mapsto ax + b \ , \ y \mapsto cy + d \ , \ y \mapsto |y|$

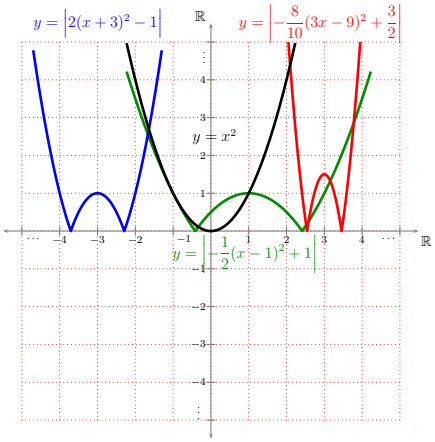

Figura 9: $y = |c(ax + b)^2 + d|$

Exemplo 2

$$\begin{array}{rcl} \text{Dom } f & = & \mathbb{R} \setminus \{0\} \\ \text{Im } f & = & \mathbb{R} \setminus \{0\} \end{array}$$

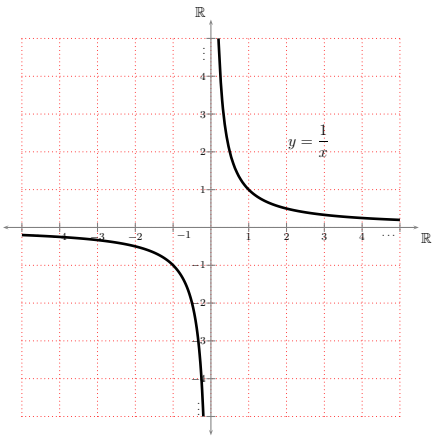

Figura 10: Modelo: $y = f(x) = \dfrac{1}{x}$

Reflexão no eixo vertical

$x \to -x$

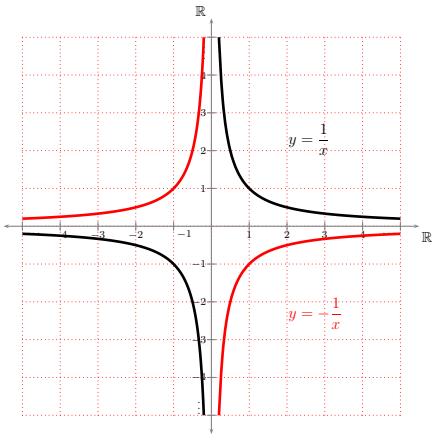

Figura 11: $y = f(-x) = \dfrac{1}{-x} = -\dfrac{1}{x}$

Reflexão no eixo horizontal

$y \to -y$

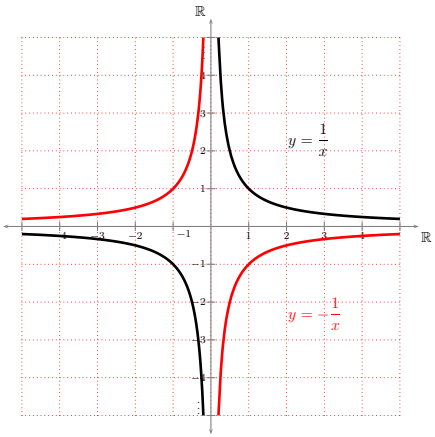

Figura 12: $y = -f(x) = -\dfrac{1}{x}$

Mudando a escala de x

$x \to ax \quad , \quad a \in \mathbb{R}$

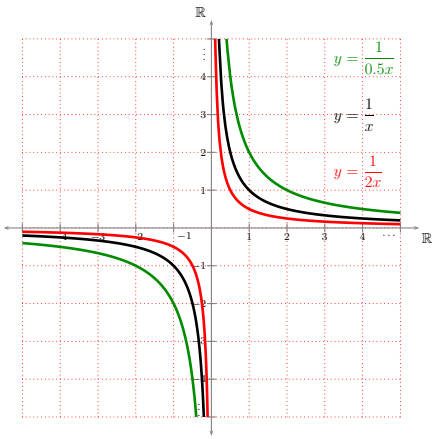

Figura 13: $y = f(ax) = \dfrac{1}{ax}$

Mudando a escala de y

$y \to cy \quad , \quad c \in \mathbb{R}$

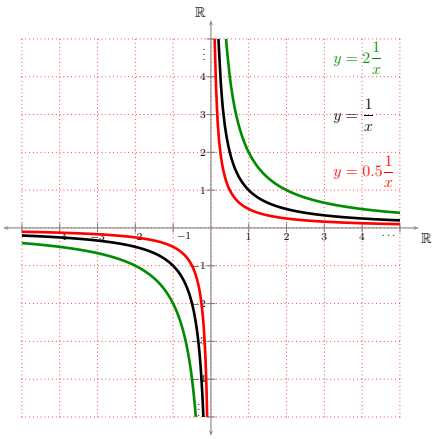

Figura 14: $y = cf(x) = c\dfrac{1}{x}$

Translação horizontal

$x \to x + b \quad , \quad b \in \mathbb{R}$

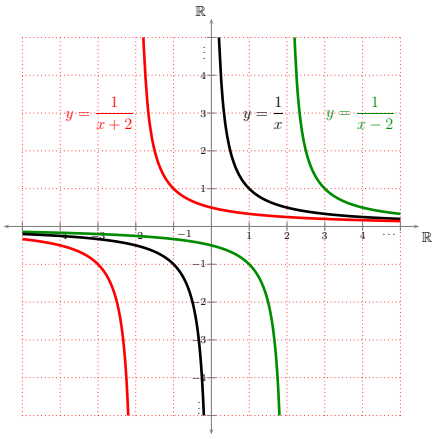

Figura 15: $y = f(x+b) = \dfrac{1}{x+b}$

Translação vertical

$y \to y + d \quad , \quad d \in \mathbb{R}$

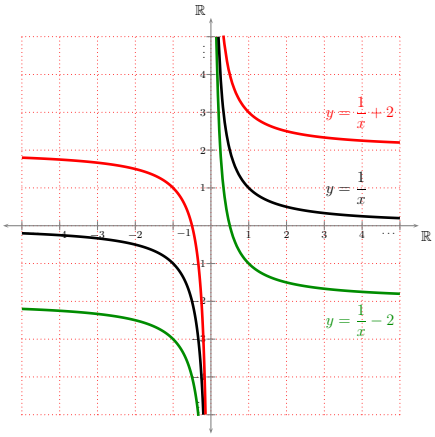

Figura 16: $y = f(x) + d = \dfrac{1}{x} + d$

Todas juntas

$x \mapsto ax + b \ , \ y \mapsto cy + d$

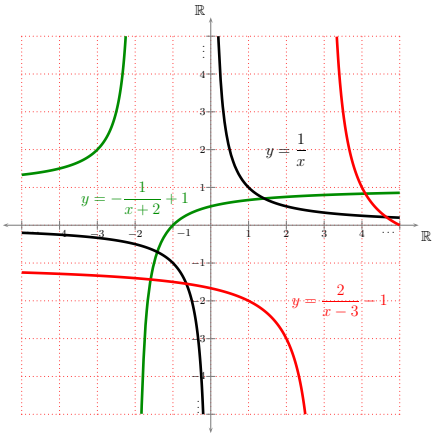

Figura 17: $y = \dfrac{c}{ax+b} + d$

Valor absoluto

$x \mapsto ax + b \ , \ y \mapsto cy + d \ , \ y \mapsto |y|$

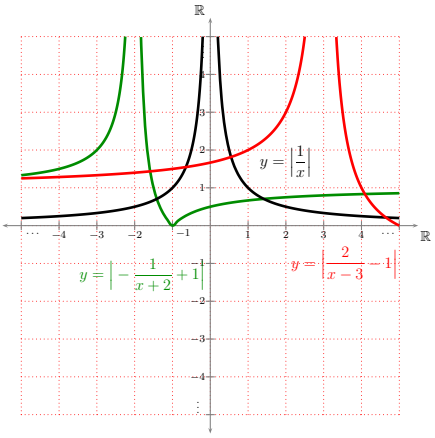

Figura 18: $y = \left|\dfrac{c}{ax+b} + d\right|$

Note-se que o domínio das funções obtidas a partir das funções modelo graficadas nas figuras anteriores muda consoante ao tipo de operação realizada.

Mais exemplos

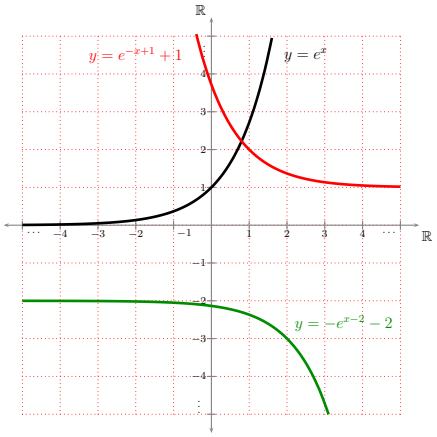

Figura 19: Modelo: $y = e^x$

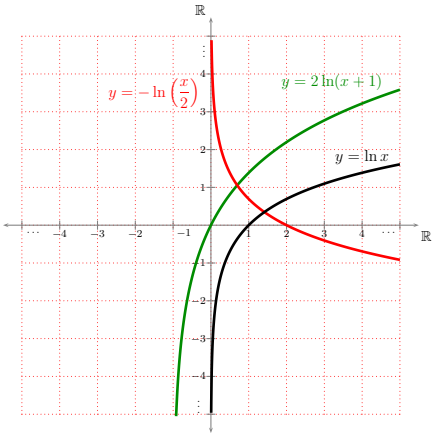

Figura 20: Modelo: $y = \ln x$

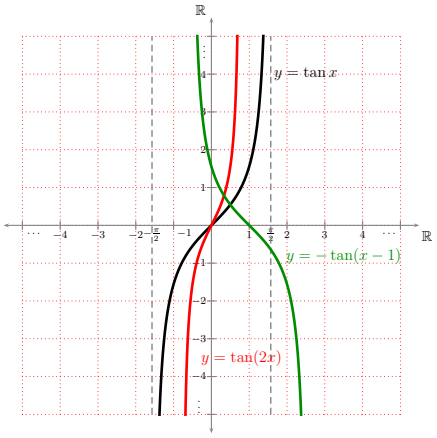

Figura 21: Modelo: $y = \tan x$ (ramo $]-\frac{\pi}{2}, \frac{\pi}{2}[$)

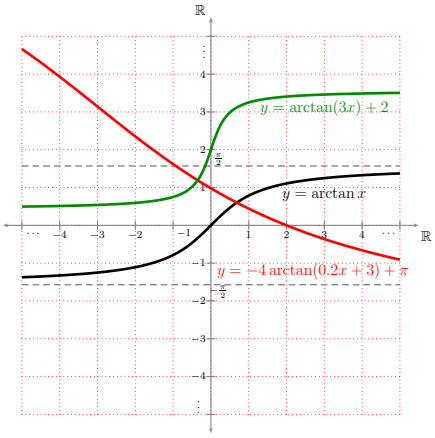

Figura 22: Modelo: $y = \arctan x$

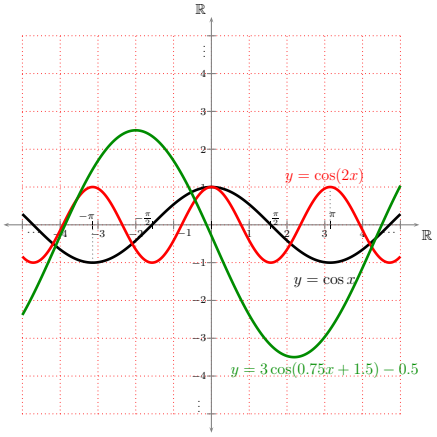

Figura 23: Modelo: $y = \cos x$

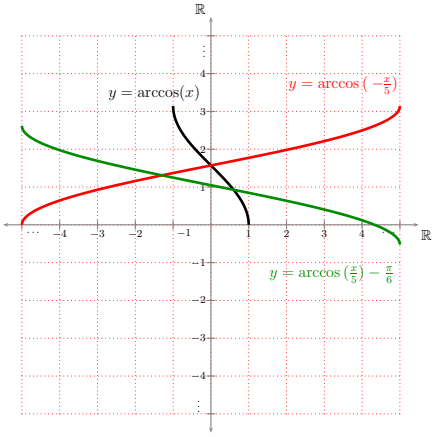

Figura 24: Modelo: $y = \arccos x$