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Área científica: II. Física dos sistemas complexos

Physics of Optoelectronic Oscillator Circuits for Communication Systems Applications

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1. Summary

Since the discover, in 1961 by Edward Lorenz, what is now known as the butterfly effect, chaos became an essential research topic in biological, chemical, physical, and social sciences. With particular interest is the use of chaotic signals for information transmission. The use of the noise-like appearance of chaotic carriers gives a very efficient way to mask messages, and is considered a promising method to improve security in communications [1,2]. Communication systems using chaotic optical carriers allow implementing steganography techniques at the physical level, which improves substantially the security of the software current encryption techniques. In such systems the receiver circuit, an identical version of the transmitter circuit, is synchronized with the chaotic emission to allow message decoding [3].

The most promising ways of producing broadband chaotic signals employ semiconductor lasers as sources of chaotic optical carriers [2]. Although semiconductor laser systems are, in general, dynamical stable, instabilities can be induced by perturbing their operation either through direct current modulation, optical injection, optical feedback, or delayed optoelectronic feedback [1,2,4]. Such perturbations correspond to the addition of new degrees of freedom to the systems and are widely proposed as methods of broadband chaotic optical signals generation [4]. However, the complexity of most of theses systems, either hybrid [1] or monolithic [2], makes practical realization very challenging.

Following recent reports on chaotic dynamics in a new type of optoelectronic circuit [5, 6], we propose an alternative laser diode system configuration capable of chaos generation based on the integration of a double-barrier quantum well (DBQW) resonant tunnelling diode (RTD) with a communications semiconductor laser diode (LD), the RTD-LD circuit/system. Such system takes advantage of the combination of RTD and LD intrinsic nonlinear dynamics, leading to substantial reduction of transmitter and receiver circuits' complexity and less restrictive operation conditions. The DBQW-RTD is a nonlinear nanoelectronic device capable to operate at room temperature as a high frequency voltage controlled oscillator (VCO), that shows wideband negative differential resistance allowing ultra-broadband electrical gain (currently up to 831 GHz [7]), that when properly perturbed shows extremely complex non-linear dynamics.

Here we propose to investigate the physics of RTD-LD systems that leads to a rich complex dynamics, including the Feigenbaum, the intermittent and the quasi-periodic routes to chaos, which are related to the period doubling, saddle-node, and Hopf bifurcations, respectively [6]. Preliminary results indicate this complex system can be an alternative to proposed optoelectronic chaotic generator systems. The investigation will comprehend the physics that leads to the system nonlinear dynamics and synchronization between two identical circuits, one acting as transmitter and the other as receiver.

Since the RTD-LD system is completely compatible with current optical communication systems, and because the combination of RTD nonlinear characteristics with LD dynamics reduces significantly operation conditions constraints of the laser diode necessary to produce high-dimensional chaos, it leads to straightforward chaos based optical communication systems implementation. The investigation will also allow a deeper insight into the theory of a new category of optoelectronic complex systems [4]

2. Literature review

2.1 Optical communications using chaotic signals

The signatures of chaotic behavior in physical systems include erratic, noise-like fluctuations in the temporal evolution of the system's variables, broadband features in the power spectrum, and extreme sensitivity to application of small perturbations to the systems' variables [8]. In the last decades, physicists have become aware that even a system that may be considered perfectly deterministic in principle can behave in a completely unpredictable in practice - deterministic chaos. Because of its intrinsically unpredictable behaviour, deterministic chaos was viewed as an undesirable characteristic in practical systems. However, it has been recognized that in certain conditions chaotic dynamics can be advantageous for a variety of applications such as information transmission [4].

Although optical communications systems are well established, providing enormous bandwidth, there is a search of technical approaches to assure the security of these systems. Among these, the use of chaotic carriers taking advantage of the aforementioned characteristics in message transmission is very promising for secure communications [1]. Optoelectronic communication systems have been explored in the last decade using chaotic signals and can have bandwidths up to tens of GHz [1,9], while chaotic electronic circuits typically have bandwidths of hundreds of kHz or less. This brings new possibilities to optoelectronic based systems of working as chaotic sources in optical chaos communication links. One of the key elements of the coupled optoelectronic systems is that electronics is coupled to a laser system, the most common source of coherent light used in optical communications. In the last decade, the optical carrier transmission of information using chaotic signals has been confirmed [9], followed by the demonstration of chaos-based communications at high bit rates using commercial fibre-optic links [1].

The laser systems usually considered in optoelectronic systems are conventionally based on semiconductor lasers. According to Arecchi's classification (based on their number of degrees of freedom) [4], these types of lasers belong to the class-B lasers. These lasers have two degrees of freedom because their dynamical behaviour can be described using a two rate equations model. According to the Poincaré-Bendixson theorem [8] they cannot be intrinsically chaotic and at least one degree of freedom must be added to produce chaos. Several ways of making semiconductor lasers chaotic have been reported and proposed over the last decades using different techniques, modulation of the cavity losses, modulation of the resonance frequency laser cavity, modulation of the injection current, external optical injection locking and delayed feedback [4]. However, these currently proposed chaotic based optical communication systems are quite complex configurations and involve a considerable number of linear and nonlinear optoelectronic devices and components.

Here we propose to investigate the complex nonlinear dynamics of a new type of optoelectronic system, taking advantage of the high-speed nanoelectronic resonant tunnelling diode (RTD) and its intrinsic gain bandwidth in series with a laser diode, recently reported in [5,6]. The output power of the laser is easily controlled through the injection current of an RTD oscillator. The chaotic dynamics of this new system can be either entirely controlled be the RTD nonlinear behaviour or present nonlinearities produced by the laser system. This means one can take advantage of both RTD and laser nonlinearities to produce high-dimensional chaos in the laser output. This system turns out to be a very rich chaos generator taking advantage of both electrical and optical components to produce instabilities and complex behaviour in the laser output.

2.2 Resonant tunnelling diodes

Semiconductor double barrier quantum well (DBQW) resonant tunnelling diodes (RTDs) are nanoelectronic semiconductor structures which utilize wave nature of electrons. Since their demonstration, in 1974 by Chang, Esaki and Tsu [10], they have become a topic of great interest, investigated, both from the standpoints of quantum physics as an important testing ground for modern theories of transport physics [10].

RTDs are excellent candidates for nanoelectronic circuit applications due to their wide-bandwidth negative differential resistance (NDR), pronounced non-linear current-voltage (*I-V*) characteristic, Fig. 1, inherent high speed, structural simplicity, relative ease of fabrication, flexible design, and versatile circuits' functionalities [11]. The RTD pronounced non-linear current-voltage (*I-V*) characteristic, Fig. 1, can be understand as follows [11]: (i) As the applied voltage across the RTD terminals is augment from zero, the current across the structure increases, mainly due to resonant tunnelling, until it reaches a local maximum (ii), the peak current, when the voltage reaches a certain value, the peak voltage. Although there is always a background current due to different transport mechanisms, in a well-designed device resonant tunnelling dominates the current flow in the *I-V* curve first positive differential resistance (PDR) branch. Continuing to increase the voltage, the current starts to drop to a local minimum (iii), the valley current, due to a reduction in resonant tunnelling, until the voltage reaches the valley voltage value: the NDR branch. In response to a further increase in the voltage the current starts to augment again, approximately in accordance with conventional rectifying diode behaviour.

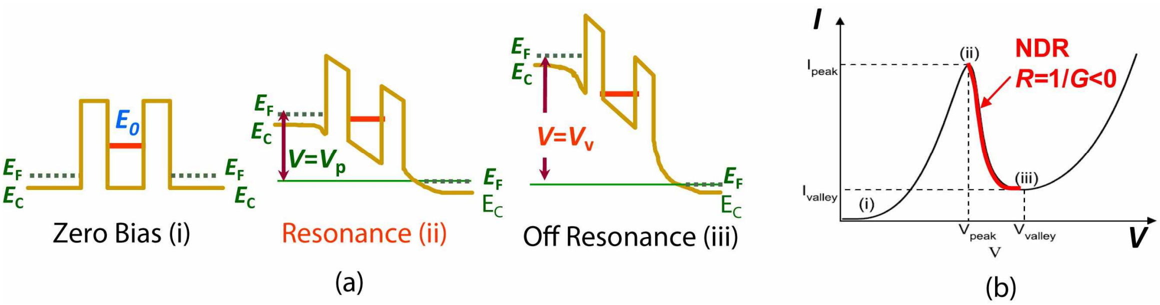

Figure 1: (a) The effect of applied bias on DBQW conduction band and energy levels. (b) Illustration of a typical RTD I-V characteristic. The tunnelling current is shown as a function of voltage. States (i), (ii), and (iii) correspond to those in (a).

Resonant tunnelling diodes (RTDs) have been proposed for high frequency signal generation, high speed signal processing, optical and microwave distribution of signals [11]. The very high speed operation arises from the extremely small size of the DBQW structure along the direction of carrier transport and the tunnelling process responsible for carrier flow. Since the NDR corresponds to electric gain it can be applied to signal generation, detection and mixing, in multi-valued logic switches at extremely high frequency, as well as in low-power amplifiers, local oscillators and frequency locking circuits, and generation of multiple high frequency harmonics, extending well into the submillimetre-wave band [12].

RTD based optoelectronic relaxation oscillators can generate constant width short electrical and optical pulses over a large range of repetition rates (up to tens of GHz), be phase-locked at the harmonic and sub-harmonic frequencies [12,13], and be easily integrated either in hybrid or monolithic configurations [13,14].

Besides the high speed potential, the negative differential resistance (NDR) makes it possible to operate RTDs as so-called functional devices, enabling circuits to be designed on different principles from those applied to conventional devices. Functional applications, such as a one-transistor static random access memory, signal processing circuits with significantly reduced number of devices and multiple-valued memory cells using RTDs have been proposed and demonstrated [15]. These functional applications are highly promising since RTDs, with their simple structure and small size, can be integrated with electronic and optoelectronic conventional devices, such as field effect and bipolar transistors, optical waveguides, lasers and photo-detectors.

Optoelectronic devices/circuits incorporating resonant tunnelling diodes/effects include light-emitting diodes, electroabsorption modulators, quantum-well lasers and resonant tunnelling injection lasers, optical two state memories [16]. The integration of RTDs with optoelectronic devices uses the benefits of the RTD nonlinearities and applies them to increase the optoelectronic systems functionalities.

3. Plan and methods

The research activities associated with the proposed optoelectronic RTD-LD system involves the study the physics that determine RTD-LD optoelectronic non-linearities in order to predict the best designs and operating parameters that leads to the production of high-dimensional chaos and robust synchronization. In what follows, we describe in detail the optoelectronic system and the synchronization methods proposed.

3.1 Optoelectronic RTD-LD system

The optoelectronic system is based on the integration of a RTD driving circuit with a laser diode. The RTD driver is coupled to the laser diode (LD) forming a directly modulated laser diode RTD-LD system. The addition of the RTD to the laser driving circuit enhances the laser diode non-linear behaviour under external perturbation which leads to higher-dimensional chaos.

The experimental RTD-LD prototype configuration is presented in Fig. 2 and consists of a hybrid integration of a laser diode and a RTD. The circuit shown in Fig. 2 is implemented on a printed circuit board and consists of a DC power supply to bias the RTD and the laser, an external signal generator to inject RF signal in the circuit, and a shunt capacitor-resistor. In Fig. 3 the current-voltage characteristics of individual and integrated devices are presented, showing that the negative resistance region is preserved in the integrated circuit.

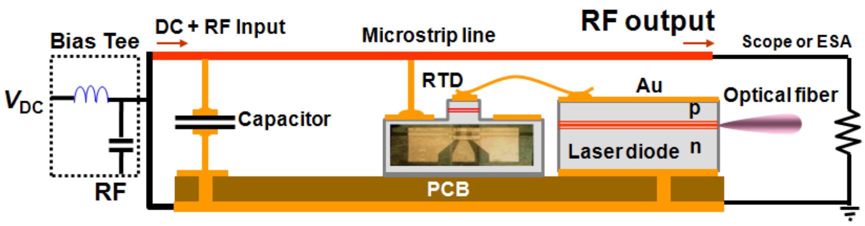

Figure 2: Representation of the electrical connections on the Printed Circuit Board (PCB). In the present configuration, the RTD is in series with the laser diode. To prevent spurious oscillations induced by the d.c. bias source circuit, V_{DC}, a shunt capacitor is connected to the RTD-LD module. The circuit RF output is used to feed an RF load. The laser output is detected using a wide bandwidth photo-detector and visualized in an oscilloscope/spectrum analyser.

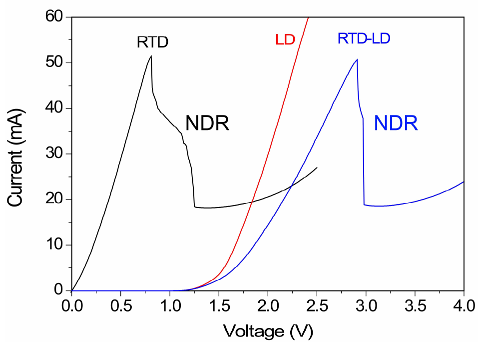

Figure 3: Current voltage (*I-V*) characteristics of the individual and integrated components, RTD, LD and RTD-LD, respectively, showing the negative resistance region (NDR) is preserved in the RTD-LD.

Figure 4 shows the equivalent electric circuit of the circuit presented in Fig 2: a laser diode driver circuit that includes the RTD, an inductor and a resistor. The capacitance in Fig. 2 corresponds to the RTD intrinsic capacitance. When not driven, the circuit produces self-oscillations - relaxation oscillations both in the optical and electrical domains.

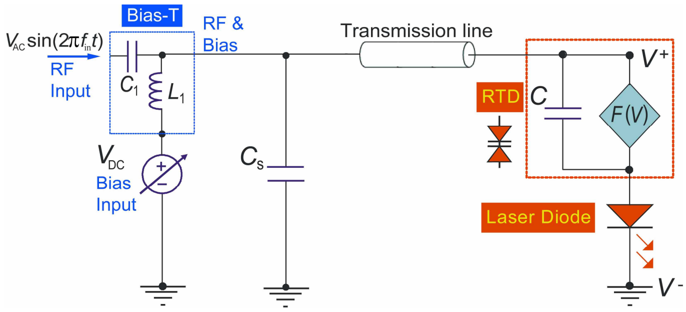

Figure 4: Equivalent electric circuit schematic of the hybrid RTD-LD optoelectronic integrated circuit represented in Figure 2. The injected time varying signals and the dc bias were supplied across the shunt capacitor circuit's input through a bias-T. The RTD is represented by its intrinsic capacitance *C* in parallel with a voltage dependent current source *F*(*V*).

When driven by periodic signals the optoelectronic circuit can display a sort of behaviours, depending of the applied signal characteristics: i) the natural oscillations can become entrained to oscillate at the same frequency as the driving force – frequency locking - Fig. 5; ii) or the natural and the applied signals can give rise to quasi-periodic signals or even chaotic behaviour in which the frequency content of the waveforms across the device dramatically changes with the characteristics of the applied signal, Fig. 6. The behaviour transition sequences include frequency division, intermittency, frequency locking and several other phenomena [5,6,17]. The circuit is capable to generate deterministic chaos, with the long-term behaviour being extreme sensitivity to small perturbations to the circuit variables.

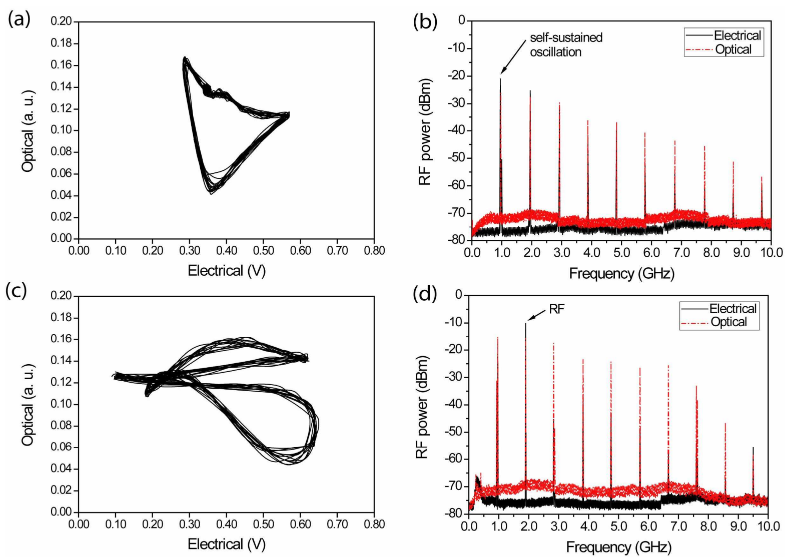

Figure 5: Experimental electro-photonic regimes for stable dynamic states. (a) Self-sustained oscillations trajectories in the *V-I* phase space around 0.953 GHz and (b) corresponding spectra. (c)-(d) synchronized phase space loops and corresponding spectra when the circuit is externally perturbed with voltage signals of 300 mV amplitude, and frequencies, f_{in}, 1.900 GHz (reported in [5]).

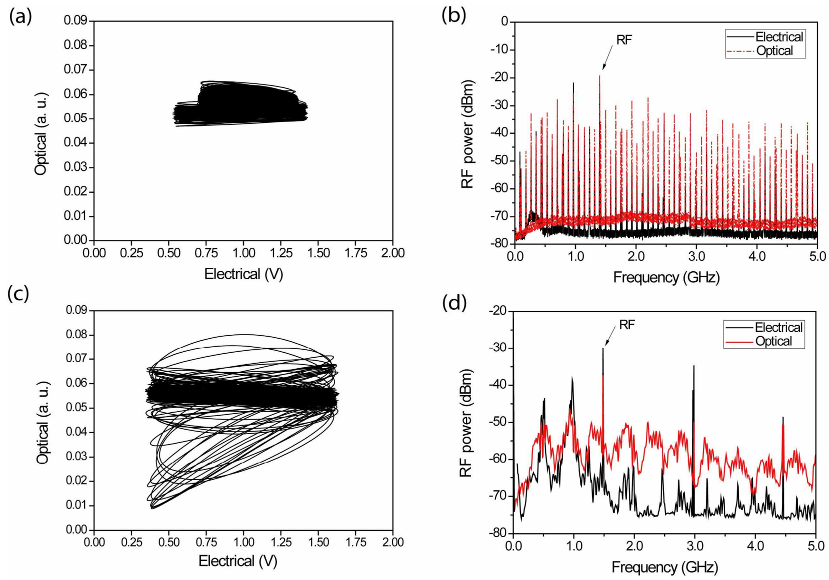

Figure 6: Experimental quasi-periodic and chaotic scenarios. (a) Quasi-periodic signal in the phase space, *V*-*I*, and (b) corresponding power spectra of detected electrical and optical outputs when f_{in}=1.440 GHz and V_{AC}=793 mV. (c) Chaotic signal in the phase space and d spectra for the operating parameters f_{in}=1.485 GHz and V_{AC}=793 mV (reported in [5]).

3.2 RTD-LD Dynamics Model

The study of nonlinear circuit oscillators has been important in the development of the theory of dynamical systems. For numerical purposes, the circuit of Fig. 4 can be modelled by the lumped *RLC* circuit configuration presented in Fig. 7.

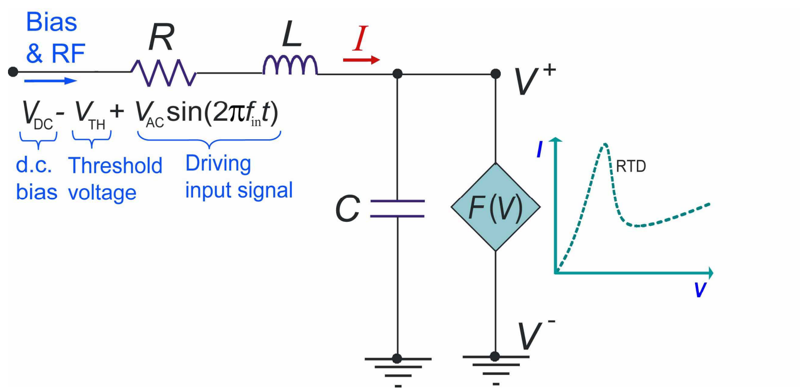

Figure 7: Lumped electric circuit of the equivalent RTD-LD circuit represented in Fig 4: a laser diode driver circuit that includes a resonant tunnelling diode, an inductor and a resistor. The laser diode is modelled using just linear elements with a combined ideal diode in series with a voltage source, V_{TH}. The capacitance corresponds to the RTD intrinsic capacitance.

By applying Kirchoff's laws (using Faraday's law) to the circuit of Fig. 7, the voltage *V* across the capacitance *C* and the current *I* through the inductor *L* are given by the following set of two first-order non-autonomous differential equations:

$$\begin{aligned} \dot{V} &= \frac{1}{C}\left[I - F(V)\right] \\ \dot{I} &= \frac{1}{L}\left[V_{DC} - RI - V + V_{AC}\sin\left(2\pi f_{in} t\right)\right] \end{aligned} \qquad (1)$$

where V_{DC} is the DC bias voltage, $V_{AC} = A \sin(2\pi f_{in} t)$, and $F(V)$ is the *I-V* characteristic of the RTD (current through the RTD-LD) given by:

$$F(V) = M \cdot \left\{ A \cdot \ln\left[\frac{1+e^{q(B-C+n_1V(t))/kT}}{1+e^{q(B-C-n_1V(t))/kT}} \right] \cdot \left[\frac{\pi}{2} + \tan^{-1}\left(\frac{C-n_1V(t)}{D} \right) \right] + H \cdot \left(e^{n_2 qV(t)/kT} - 1 \right) \right\} \qquad (2)$$

where *e* is the electric charge, *K* the Boltzmann constant and *T* the Temperature (K), *A*, *B*, *C*, *D*, *H*, n_1, n_2, *M* are fitting parameters, having at the same time a well-defined physical interpretation: *A* and *B* are related, among other factors, with resonance width and Fermi level energies, and allow to adjust RTD peak current; *C* and n_1 determine essentially the RTD peak voltage, correlated with the energy of the resonant level relative to the bottom of the well and with the transmission coefficient; finally, *D* is related with the resonance. After some algebra, we found the system of equations (1) correspond to the generalized Liénard's system under external periodic injection given by:

$$\ddot{V} + H(V)\dot{V} + G(V) = V_{AC}\sin(2\pi f_{in} t) \qquad (3)$$

where $G(V)$ is a nonlinear force, $H(V)\dot{V}(t)$ is the damping factor:

$$\begin{aligned} H(V) &= \frac{R}{L} + \frac{1}{C}\dot{F}(V) \\ G(V) &= \frac{V}{LC} + \frac{R}{LC}F(V) - \frac{V_{DC}}{LC} \end{aligned} \qquad (4)$$

In order to correlate the response of the laser diode to its physical parameters we use rate equations to describe the transient behaviour of the laser. The laser rate equations describe the interaction between electrons and photons within the laser cavity that determine the laser nonlinearity.

Assuming the laser oscillates in a single mode, the population inversion is homogeneous and the gain is linear, the rate equations for the photon *S* and injected carrier density *N* in the active region are:

$$\begin{aligned} \dot{N} &= \frac{I}{q\vartheta} - \frac{N}{\tau} - g_0\left(N - N_0\right)\frac{S}{1+\varepsilon S} \\ \dot{S}(t) &= g_0\left(N - N_0\right)\frac{S}{1+\varepsilon S} - \frac{S}{\tau_p} + \frac{\beta N}{\tau}, \end{aligned} \qquad (5)$$

where *I* is the current through the laser diode, here given by Liénard's equation (4) and the bias current, *q* is the electron charge, ϑ is the active region volume, τ_n and τ_p are the spontaneous electron and photon lifetimes, respectively; β is the spontaneous emission factor; g_0 is the gain coefficient, N_0 is the minimum electron density required to obtain a positive gain and ε is the value for the nonlinear gain compression factor [14]. The model also considers the inclusion of the spontaneous emission effect β and the nonlinear gain compression factor ε in order to take in account laser nonlinearities.

The RTD-LD system is then described by the coupling between the laser rate equations and the RTD Liénard's oscillator. In this investigation we intended to understand the physics that determine the instabilities occurring in the laser diode and the contribution of laser intrinsic factors to the full system dynamics. The effects of nonlinear gain reduction and spontaneous emission factors will be investigated to find their influence in the appearance and disappearance of new routes to chaos.

3.3 Synchronization of chaos

The key for chaos based communications is chaos synchronization [18]. In 1990 Pecora and Carroll [3] reported that chaotic systems possess a self-synchronization property showing that certain subsystems can be synchronized by linking them with common signals. In general the synchronization of a transmitter-receiver system can be achieved by injection, unidirectional or mutual coupling synchronization. They considered the situation of unidirectional driving, in which one has a couple of transmitter-receiver systems such that a signal from the transmitter is injected into the receiver in such a way that both systems become synchronized. They first considered the case of synchronizing two exact replicas of a given system (homogeneous driving) starting with different initial conditions. Then, they also showed that synchronization is robust to small perturbations on the parameters of the transmitter or receiver systems. This is an important result for experimental settings, where one does not usually have two exact replicas of a chaotic system. This situation is usually referred to as inhomogeneous driving.

Following Pecora and Carroll work it is possible to construct a synchronization scheme using identical RTD-LD emitter/receiver systems based on optical injection phenomena. This technique is usually called complete chaos synchronization [4,18]. The chaos synchronization occurs after the receiver receives a chaotic signal from the transmitter when the transmitter and receiver are divided into several subsystems. In this case, the time lag of the signal in the receiver system is defined by time τ, which is the transmission time of signal from the transmitter to the receiver. In general, this type of chaos synchronization in laser systems is achieved by optical injection locking and amplification of signals form the transmitter to the receiver. This is a well-known phenomenon of injection-locking in laser systems and can be used to study the synchronization performance of RTD-LD operating in the chaotic regime.

The application of this chaos synchronization scheme to the novel RTD-LD system will provide a deeper insight into the transmission and synchronization of chaotic signals in these new types of optoelectronic systems. The investigation in detail of the synchronization scheme of two chaotic RTD laser systems can produce an innovative approach in securing data communications by message encryption at the physical level offering a certain degree of intrinsic privacy, which can complement both classical software-based and other physical-based cryptography systems.

4. Tasks description and expected results

Following the guidelines presented in the previous section which describes the optoelectronic system and synchronization schemes, the research activities aim the investigation of the optoelectronic complex system that has the potential to implement novel optical communication systems functions, especially optical communications using chaotic signals. The application mainly consists to investigate optoelectronic RTD-LD systems that integrate a laser diode with a RTD to obtain novel optoelectronic electrical-optical converters and sources of electrical and optical chaotic carriers.

The work proposal is organized to achieve two major goals: first, investigation of models capable of describing more accurately RTD-LD optoelectronic non-linearities in order to predict the best designs parameters to achieve robust synchronization; second, demonstration of chaotic communication links employing synchronized RTD-LD based circuits.

4.1 Task 1

The research will encompass both the formation and characterization of the nonlinear oscillator and generation of the optical signal, namely:

a) formulation of numerical physical model of the RTD-LD optoelectronic systems;

b) study of RTD-LD nonlinear dynamics and chaotic capabilities considering the following RTD and LD physical parameters (using circuit parameters as control parameters):

> **b1) RTD:** influence of resonance width, Fermi level energies, (related with RTD peak current); and energy of the resonant level relative to the bottom of the well and with the transmission coefficient (related with the RTD peak voltage), in the chaotic dynamics of the system;
>
> **b2) LD:** influence of nonlinear gain reduction, spontaneous emission factor, bias current and current modulation in the appearance and disappearance of new routes to chaos in the system;

c) determination of the physical parameters to achieve robust chaotic optical sources and the synchronization restrictions introduced by the addition of data;
d) investigation of several information encoding techniques such as chaos masking, chaos modulation and chaos shift keying will be considered.

The computing tools to be used include, “Mathematica”, “Matlab”, and “Spice” – the general purpose analog circuit simulator. The chaotic high-dimensional dynamics of RTD-LD systems will be analyzed via trajectories diagrams (in the cylindrical space, in the plane space and Poincaré maps), synchronization/bifurcation maps (*n*-dimensional/Arnold tongues and circle maps/Devil staircases) and chaos characterization using Lyapunov spectrum, dimension and entropy analysis.

This task should involve the collaboration with the Instituto de Microelectrónica de Sevilla, IMSE-CNM, Universidad de Sevilla, Seville, Spain (Dr. J. M. Quintana and Dr. M. J. Avedillo).

4.2 Task 2

Once completed task **1**, the system will be tested against the theoretical models in a test-bed demonstration. This task should involve the collaboration with the Department of Electronics and Electrical Engineering, University of Glasgow, United Kingdom (Drs. T. J. Slight, L. Wang, E. Wasige and C. N. Ironside). This task includes:
a) optimization of RTD-LD circuits capable of chaotic generation;
b) implementation of schemes of synchronization of two circuits and optical link;
c) experimental determination of the feasibility of chaos-encoded communication systems with transmitter-receiver RTD-LD oscillators;
d) investigation the necessary chaotic synchronization conditions and methods for transmission of data through direct chaotic carrier data encoding.

If achieved the main goals of **task 2**, we expect to have a significant impact in the state of the art of optoelectronic systems in general and chaos based systems in particular, namely:

- demonstrate a new method of transmission based on chaos;
- improve the state-of-the-art of existing methods of chaotic communication systems based on chaos synchronization towards component integration and miniaturization through the introduction of chaotic optical oscillators based on RTD-LD.

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