Population growth in ideal habitats

How does a population grow when colonizing an habitat under ideal physical and biological conditions ?

The muskox

Original distribution: North America, Greenland

Depleted by hunting from 1700 to 1850

Last individuals in Alaska: 1850-60

Nunivak Island

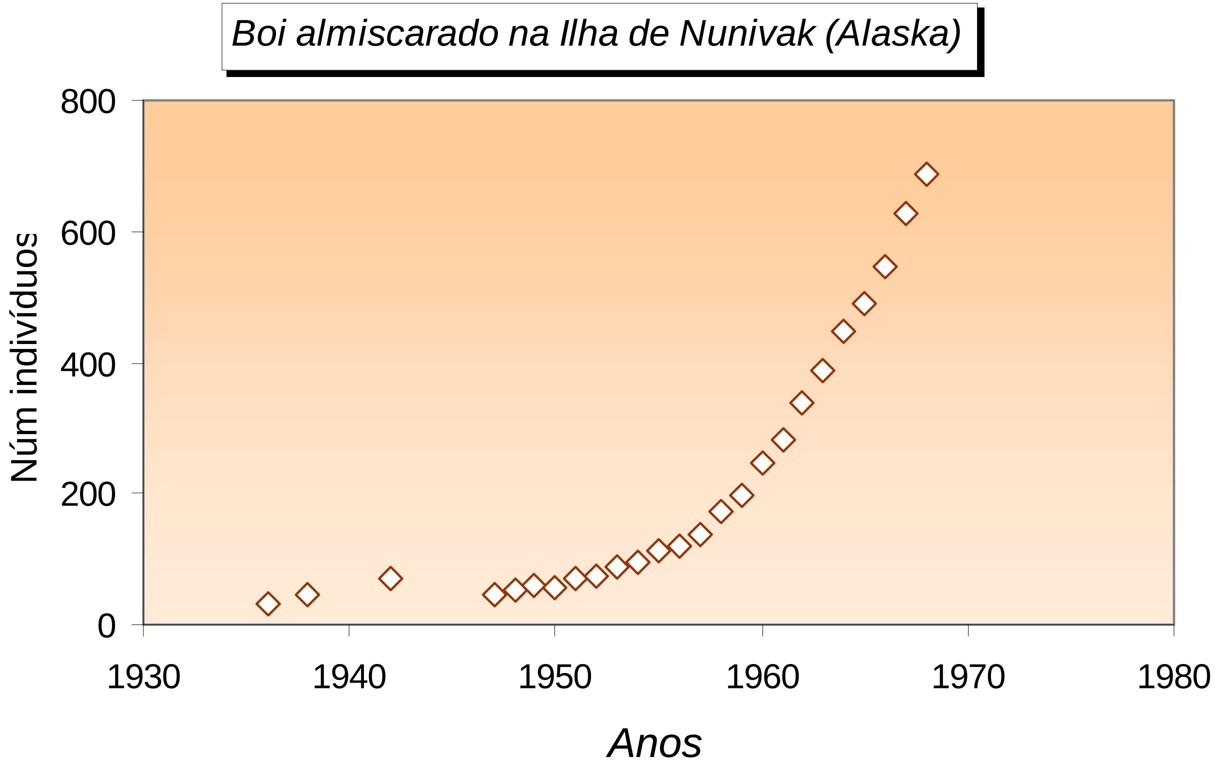
Nunivak Island

31 animals, 1936

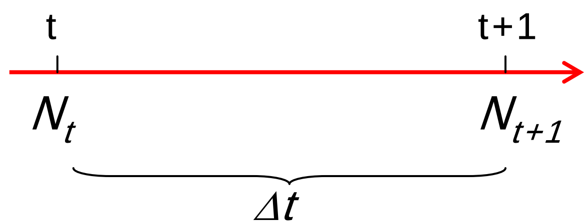
Geometric growth

Muskox
(Ovibos moschatus)

Initial population at the Nunivak reserve: 31 individuals



Measures of variation in N



$\Delta N > 0$ growth
$\Delta N = 0$ no change
$\Delta N < 0$ decline

$\Delta N = N_{t+1} - N_t$ *Absolute variation*

$$\frac{N_{t+1} - N_t}{\Delta t} = \frac{\Delta N}{\Delta t}$$ *Mean variation over* $\Delta t \equiv$ *variation time*$^{-1}$

$$\frac{1}{N_i}\frac{\Delta N}{\Delta t}$$ *Mean relative variation* $\equiv$ *% variation*

Finite rate of increase

$$\frac{N_{t+1}}{N_t} = \lambda$$

λ = finite rate of increase

What happens if λ remains constant ?

$$N_{t+2} = \lambda \; N_{t+1}$$

$$N_{t+2} = \lambda \lambda \, N_t = \lambda^2 \; N_t$$

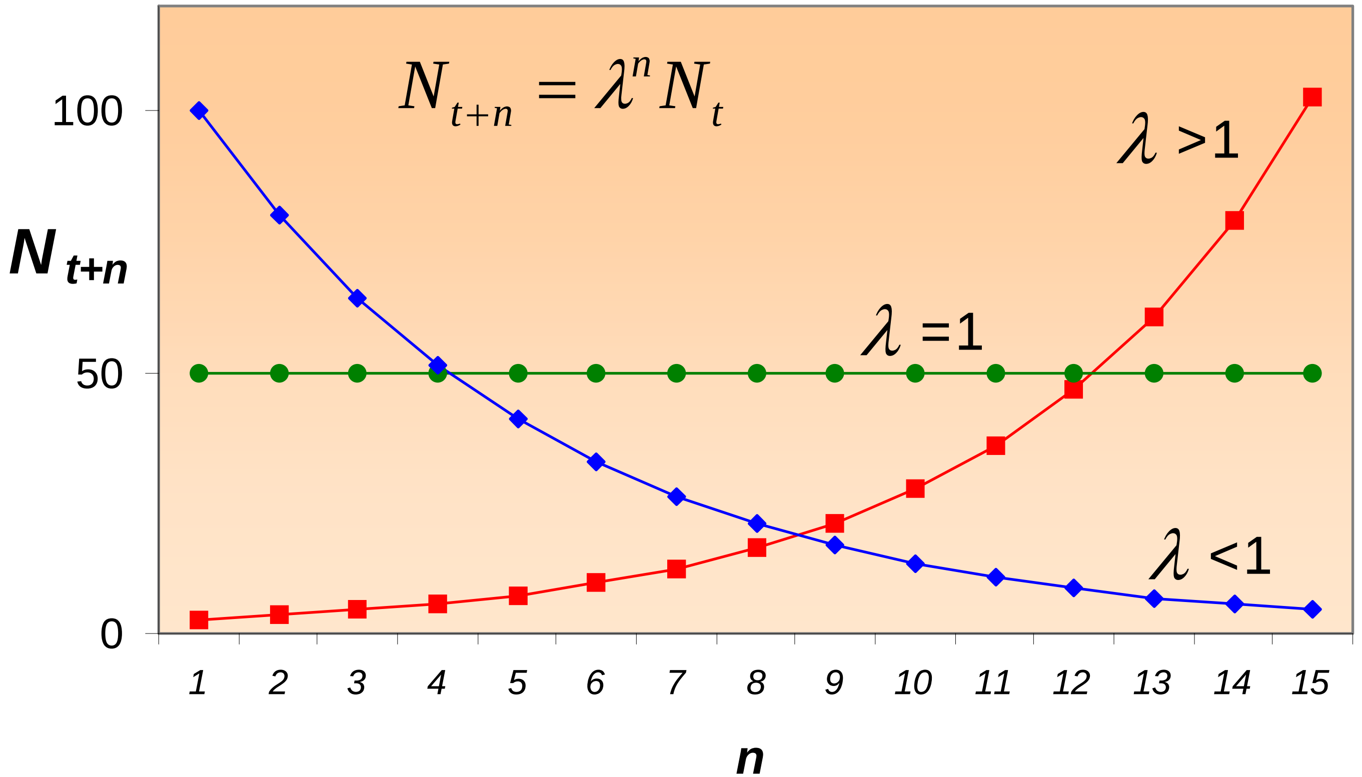
$$N_{t+3} = \lambda \; N_{t+2} = \lambda^3 \, N_t$$

…

$$N_{t+n} = \lambda^n \; N_t$$

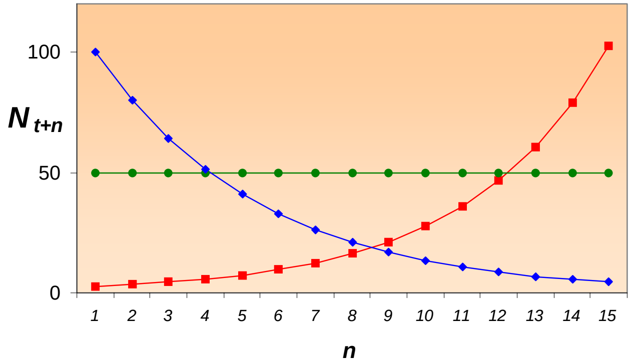
$$N_{t+n} = \lambda^n N_t$$

Geometric growth



Do you recognize the muskox here ?

$$N_{t+n} = \lambda^n N_t$$



May λ remain constant ?

$$\lambda = \frac{N_{t+1}}{N_t}$$

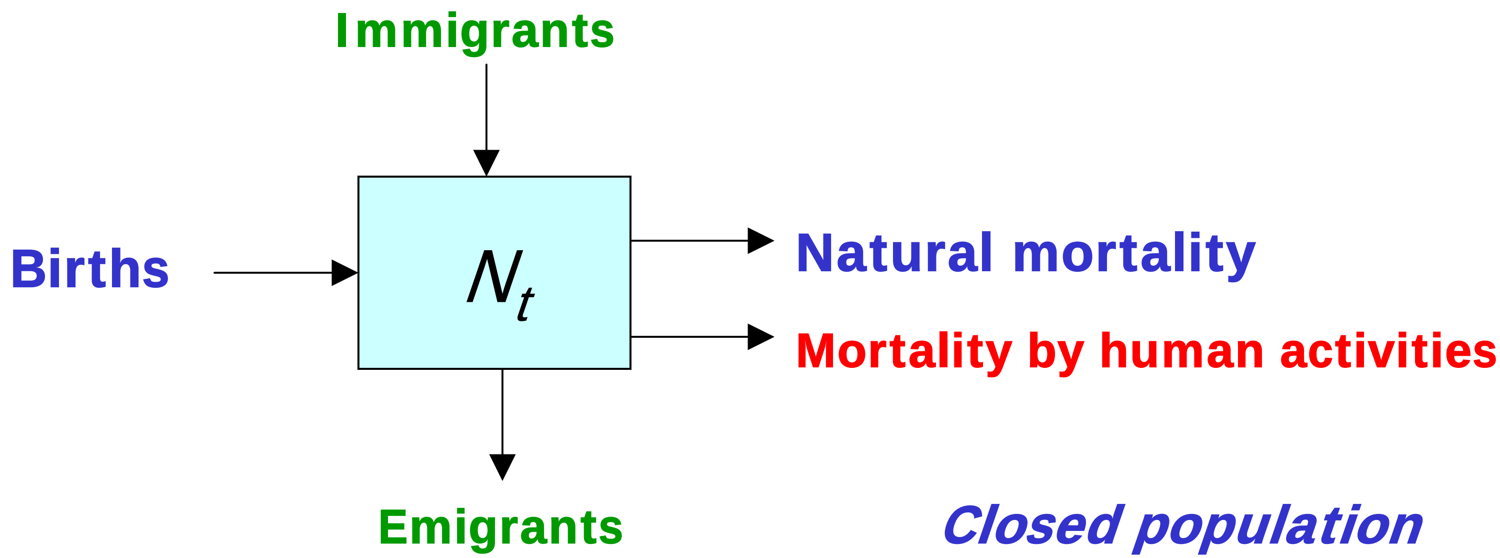
Contribution of each individual in t, for the population in t+1

Biological meaning of λ ?

What is the biological meaning of λ ?

Are newborns envolved ? deaths ? both ?

Why does N_t change ?

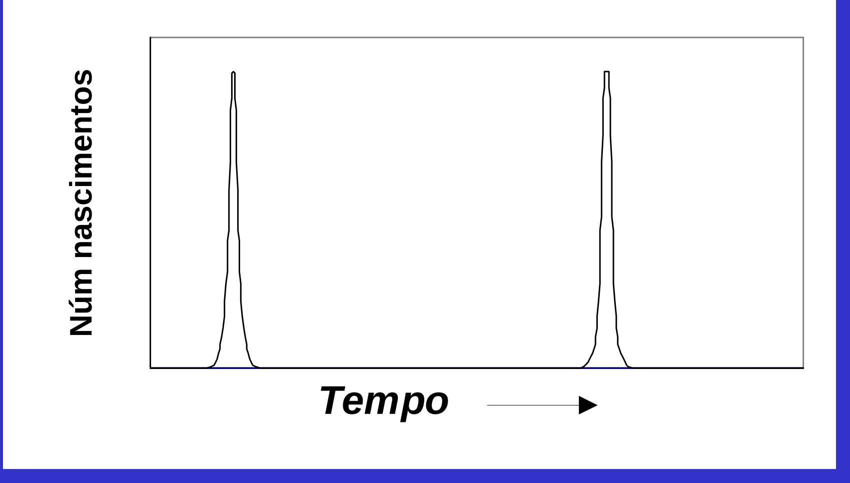


Closed population

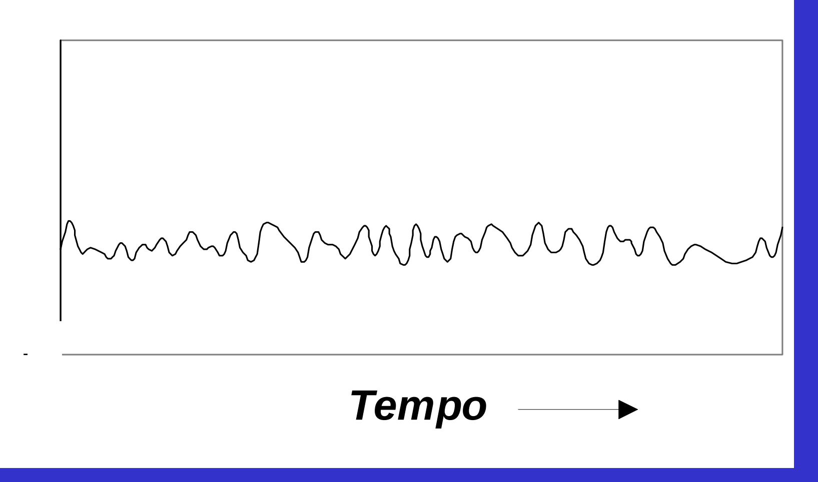
Open population

$$N_{t+1}=N_t-D_t+B_t$$

Reproduction timing

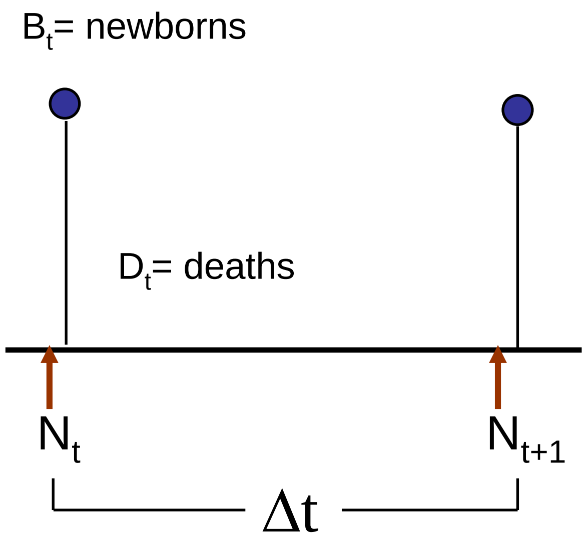


Seasonal breeding



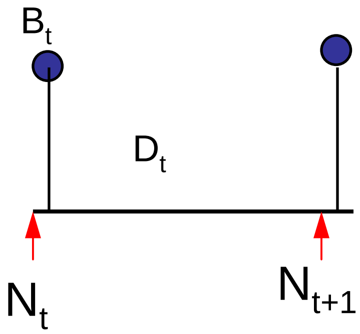
Continuos breeding

Census and reproduction in seasonal breeders



Pre-breeding census

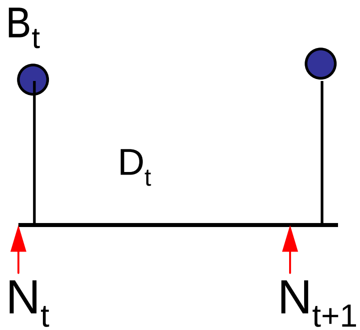
Survival rate



$$S_t = \frac{N_t + B_t - D_t}{N_t + B_t} = \frac{N_{t+1}}{N_t + B_t}$$

Birth rate

$$b_t = \frac{\textit{Number of newborns}}{\textit{Number of parents}}$$



$$b_t = \frac{B_t}{N_t}$$

Biological meaning of λ

remember $$\lambda_t = \frac{N_{t+1}}{N_t}$$

Substituting N_{t+1}

Using:

$$S_t = \frac{N_{t+1}}{N_t + B_t} \quad \therefore \quad N_{t+1} = S_t\left(N_t + B_t\right)$$

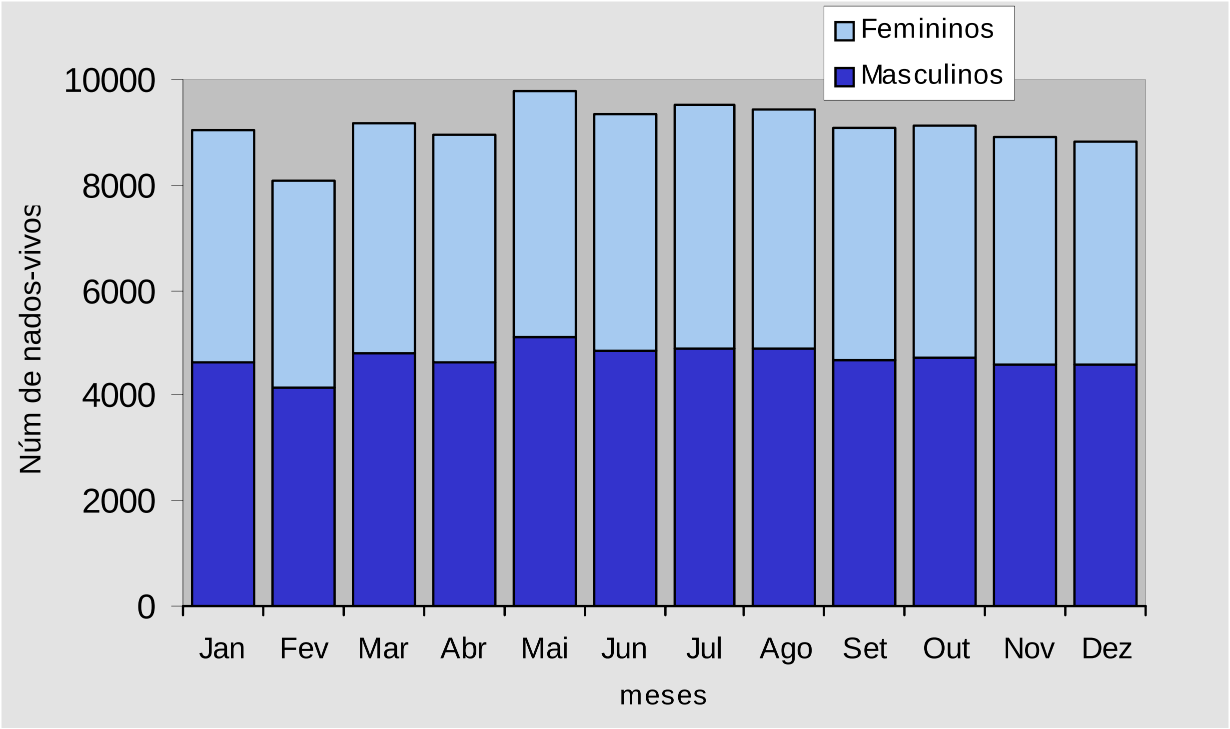
We get:

$$\lambda = S_t\left(1 + b_t\right)$$

Birth rate

Survival rate

Newborns in Portugal, 1994, INE 1995



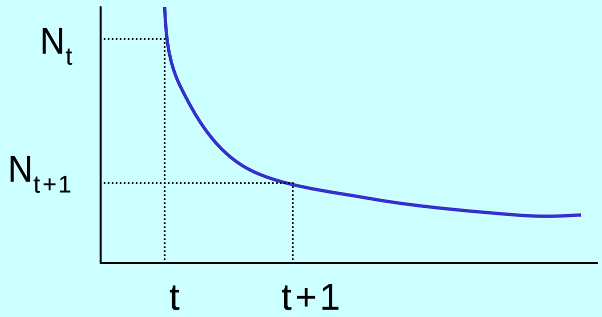
Continuous reproduction

N changes continuously !

Any time interval Δ t= [t, t+1] will be arbitrary

Remember mean variation :

$$\frac{N_{t+1} - N_t}{\Delta t} = \frac{\Delta N}{\Delta t}$$



$$\lim_{\Delta t \to 0} \frac{\Delta N}{\Delta t} = \frac{dN}{dt}$$ = *Instantaneous* variation at t

Instantaneous variation

Instantaneous variation at time t:

$$\frac{dN}{dt} = B_t - D_t$$

Instantaneous rates,

Birth rate =

$$\frac{newborns}{parents} = \frac{B_t}{N_t} = b_t$$

Mortality rate =

$$\frac{deaths}{population} = \frac{D_t}{N_t} = d_t$$

Instantaneous rate of growth

$$\frac{dN}{dt} = N_t b_t - N_t d_t = N_t \underbrace{(b_t - d_t)}_{r}$$

r

Instantaneous rate of growth

(Malthusian parameter)

$$\frac{dN}{dt} = rN$$

r units:
Individuals per individual per unit time

Given an initial N_t what is $N_{t+\Delta t}$?

Solution

$$\frac{dN}{dt} = rN$$

Ordinary differential equation of 1st degree

Assuming r is constant,

Solution, by separable variables:

Parameter

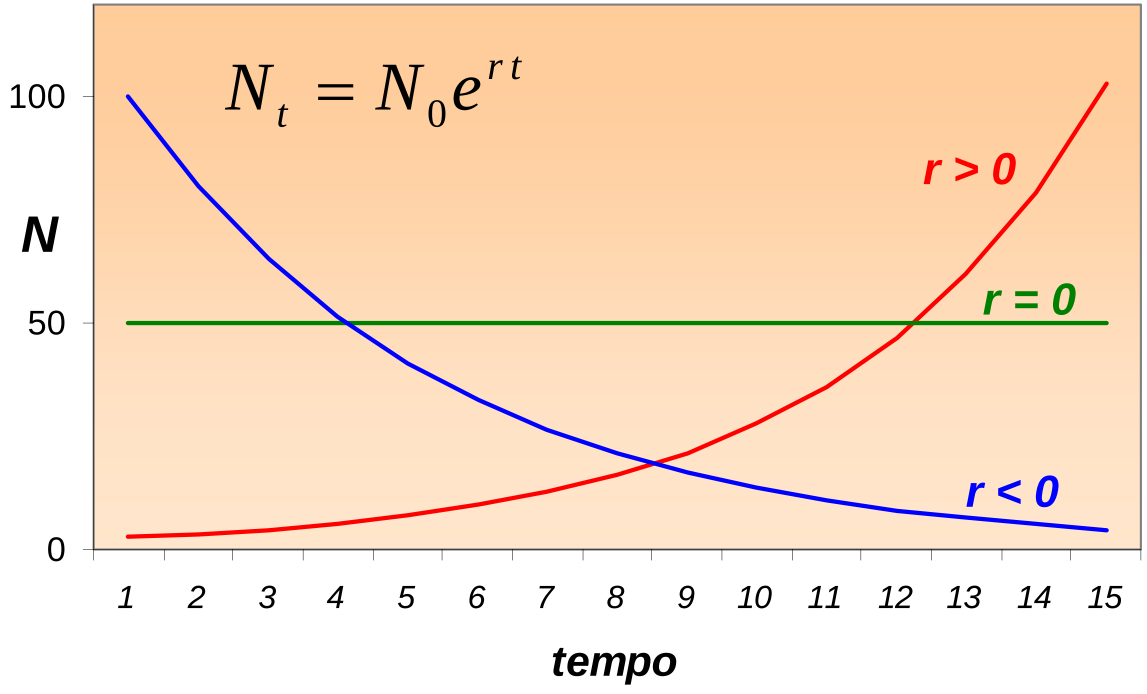
$$N_{t+\Delta t} = N_t \, e^{r\,\Delta t}$$

Independent variable

For any Δ t

Dependent variable

Exponential growth



Unregulated growth

Discrete time: $$N_{t+1} = N_t \lambda$$

Continuous: $$N_{t+\Delta t} = N_t e^{r \Delta t}$$

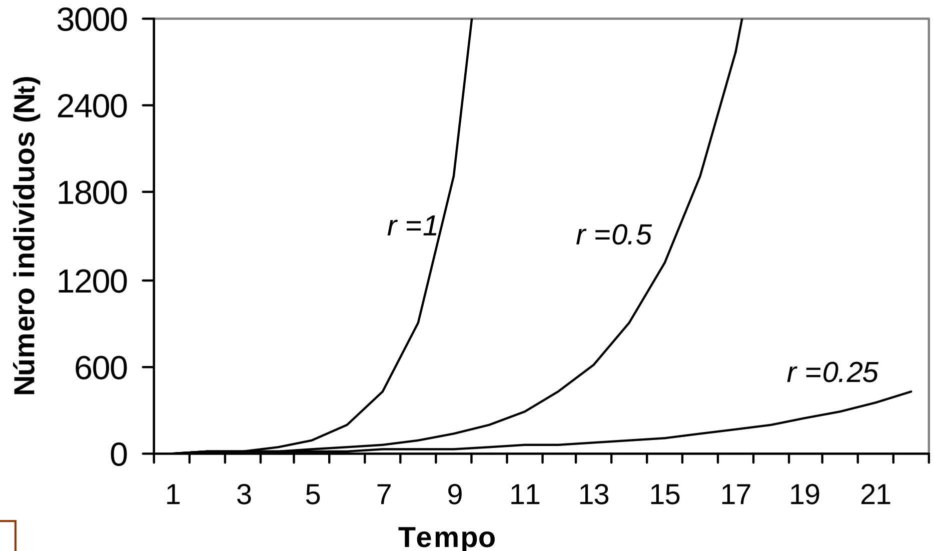
If λ *applies to the time interval* $\Delta t=1$,

Relationship between *instantaneous rate of growth* *and* *finite rate of increase*

$$e^r = \lambda$$

Unregulated growth cannot last long

$$N_{t+\Delta t} = N_t \, e^{r\,\Delta t}$$



r= 1 year $^{-1}$
N_t *= 10 initial individuals*
Δt *= 10 years*

$N_{t+10} = 10\,e^{1 \times 10}$ = *220 265 individuals*

Survival and reproduction depend upon N_t

$$N_{t+\Delta t} = N_t \, e^{r \Delta t} = N_t \, e^{(b-d)\Delta t}$$

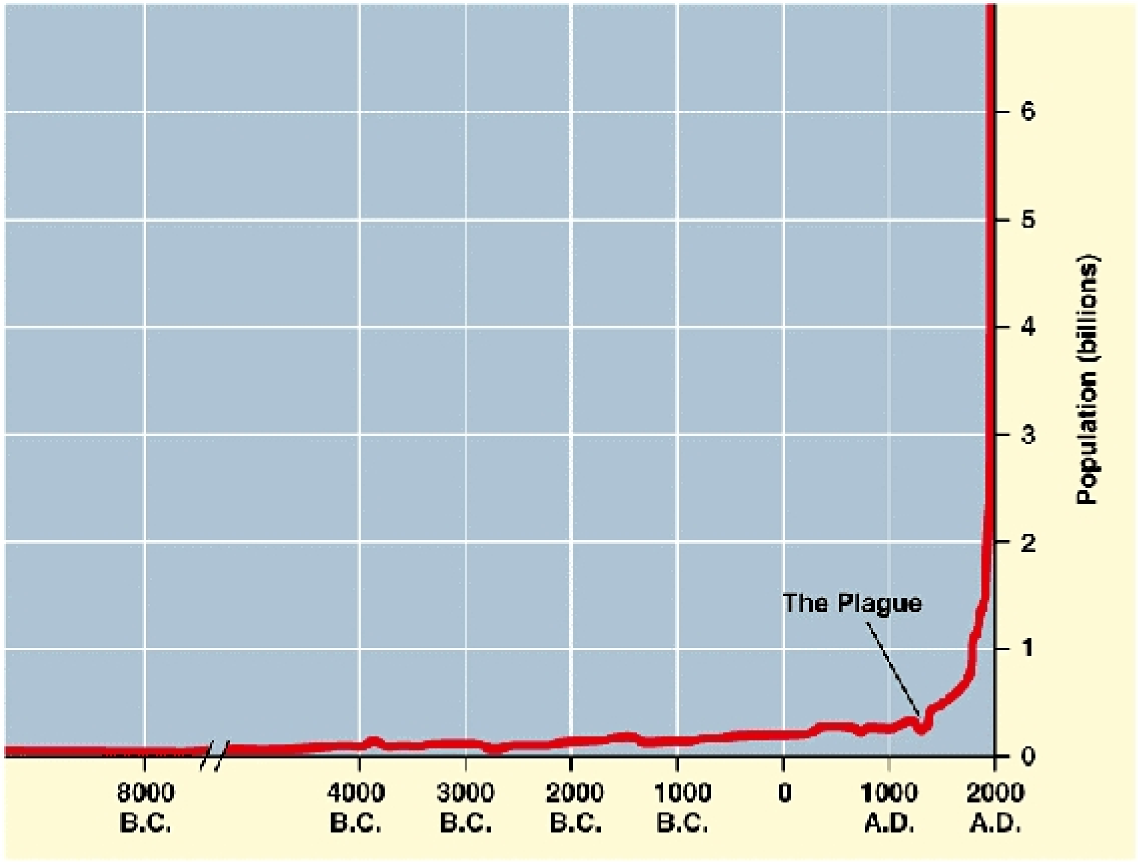
Survival, birth rate = f (N_t)

What good is the unregulated growth model if it does not apply to most populations ?

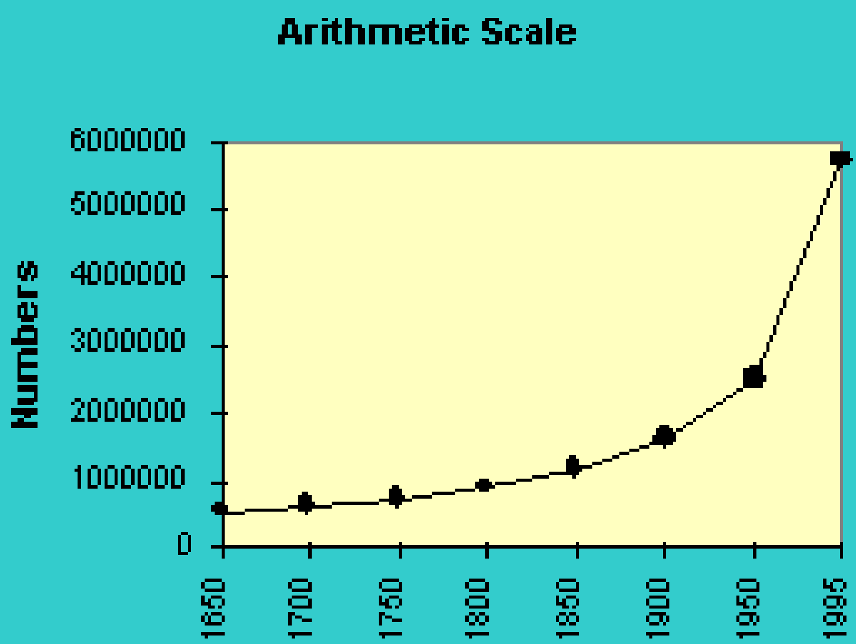
1. *Illustrates the consequence of assuming constant survival and birth rates*
2. *The model describes the initial stages of population growth, showing the enormous potential of populations to grow*
3. *It is a good starting point for the introduction of other components that confer greater realism to population growth*

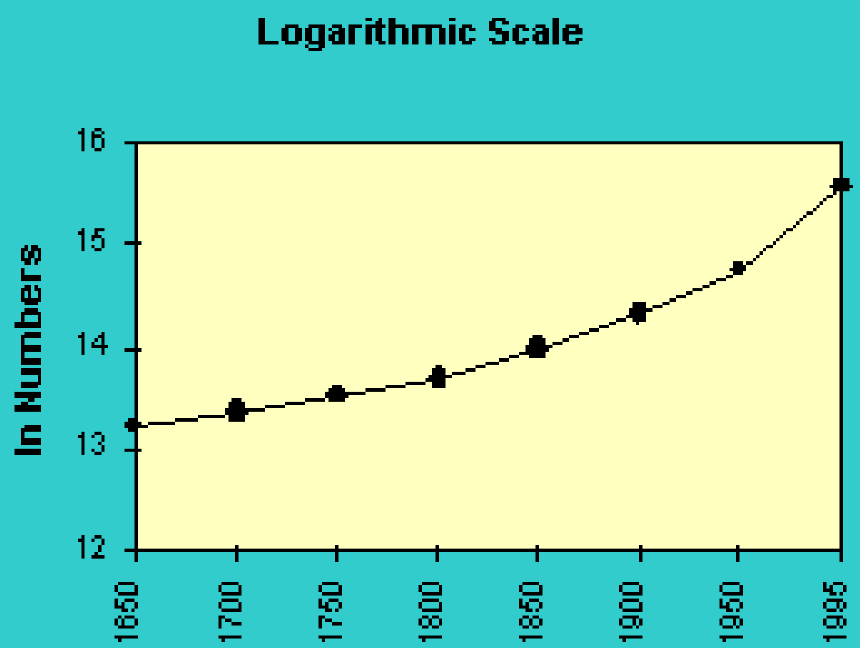
Human population 1

Figure 52.21 Human population growth



Human population 2





Source: Demographic yearbook. Annuaire démographique. New York Dept. of Economic and Social Affairs, Statistical Office, United Nations

b_t and d_t in an exponential population

Figure 52.22 Changes in birthrates and death rates in Sri Lanka