Summer Workshops at CAUL, 2013

1 Algebraic structures and semigroups

Unary FA-presentable algebraic and combinatorial structures

Alan Cain, Universidade do Porto

An FA-presentation (also called an automatic presentation) is a description of a relational structure using regular languages. The concept of FA-presentations arose from computer scientists' need to extend finite model theory to infinite structures. Informally, an FA-presentation consists of a regular language of abstract representatives for the elements of the structure, such that each relation (of arity n, say) can be recognized by a synchronous n-tape automaton. An FA-presentation is unary if the language of representatives is over a 1-letter alphabet.

In this talk, I will describe recent work with Nik Ruskuc and Rick Thomas on algebraic and combinatorial structures that admit unary FA-presentations. In particular, I will describe (1) what is known about unary FA-presentable semigroups, including progress towards a classification, and (2) a new diagrammatic technique for reasoning about unary FA-presentations, and applications to classification results for unary FA-presentable combinatorial structures of certain types, such as trees and partial orders.

Infinite monoids as geometric objects

(This is joint work with Mark Kambites)

Robert Gray, University of East Anglia

I shall describe ongoing joint work with Mark Kambites on the development of geometric methods for finitely generated monoid and semigroups. We study a notion of quasi-isometry between spaces equipped with asymmetric, partially defined distance functions (so called, semimetric spaces) and hence between finitely generated semigroups and monoids via their directed Cayley graphs. I shall give an overview of some basic concepts and results from this theory, and show how these ideas may be applied to investigate quasi-isometry invariants of finitely generated monoids.

Conjugacy in semigroups: a rewriting perspective

(This is a joint work with J. Araújo and J. Konieczny)

António Malheiro, Universidade Nova de Lisboa & CAUL

The conjugacy relation plays an important role in group theory. I will describe some of the most well know generalizations of the notion of conjugacy into the semigroup theory, and also some of the basic results. A new notion of conjugacy is presented to overcome some difficulties presented by the other generalizations. I will give special emphasis on results about conjugacy within classes of semigroups defined using rewriting systems.

Inverse semigroups and groupoids via quantales

Pedro Resende, Instituto Superior Técnico

Several relations between inverse semigroups and topological groupoids are known and well documented in the literature. Many of them can be placed within a general equivalence between inverse semigroups (or inverse semigroup actions) and their tale groupoids of germs, where the germs can be described, for instance, in terms of certain kinds of filters of the inverse semigroups. In this talk I describe how these constructions actually have two facets that can be dealt with in two separate stages, provided one is ready to replace topological spaces by locales (of whose theory I will give an overview). The first stage is concerned with creating, in a point-free way, the algebraic structure of a groupoid from that of an inverse semigroup, whereas the second one deals with the groupoid topology itself. It is only at the second stage that germs appear. What this means is that, instead of working directly with topological groupoids, the first stage relates inverse semigroups to localic groupoids (= groupoids in the category of locales), whereas the second stage is essentially a consequence of obtaining the spectrum of a locale according to general locale theory: the germs are the points. This separation of concerns yields an axiom-of-choice-free theory at the localic level, and it allows for a clearer understanding of the purely algebraic aspects, namely as regards a third actor that comes into play: the relation between inverse semigroups and localic tale groupoids is mediated by a class of quantales known as inverse quantal frames, which arise as completions (with respect to arbitrary joins) of the inverse semigroups and which can also be thought of as being groupoid "topologies" equipped with multiplication and involution obtained pointwise from those of the groupoids. (Alternatively, they are convolution algebras of the groupoids, constructed with locale-valued functions instead of field-valued ones.) Moreover, this theory can be extended to more general groupoids by similarly generalizing the class of quantales.

2 Categorical Representations in Algebra

On (co)complete spaces

Dirk Hofmann, CIDMA & Universidade de Aveiro

Employing a formal analogy between order sets and topological spaces, over the past years we have investigated notions of cocompleteness for topological, approach and other kind of spaces. In this new context, the down-set monad becomes the filter monad, cocomplete ordered set translates to continuous lattice, distributivity means disconnectedness, and so on. However, in contrast to the ordered case, the subsequent development of the notion of completeness cannot be seen as the dual image of cocompleteness; and in this talk we will explore this path:

- We describe our general framework, namely that of a topological theory $\mathcal{T}$, consisting of a monad, a quantale $\mathcal{V}$ and an algebra structure on $\mathcal{V}$. The associated notion of $\mathcal{T}$-category embodies ordered, topological, metric and approach structures. We recall succinctly the main constructions and results, in particular the important notions of distributor and representable category.
- We construct and study the "upset" monad (which turns out to be the Vietoris monad in the case of topological spaces).
- We introduce a notion of weighted limit similar to the classical notion for enriched categories, and show that the algebras for this monad are the complete categories (i.e. admit all weighted limits) and the algebra homomorphism are precisely the limit preserving functors;
- however, unlike for ordered sets, a limit preserving functor between complete categories need not be right adjoint.
- We characterise the right adjoint functors between complete categories as precisely the "downwards open" limit preserving maps;
- We explain that the statement "X is totally cocomplete iff X^{op} is totally complete" specialises to O. Wylers characterisation of the algebras of the Vietoris monad on compact Hausdorff spaces;
- We describe the Kleisli category of this monad and observe how the notion of an Esakia space arises naturally in this context via splitting idempotents.

Actions and semidirect products in algebraic categories

Manuela Sobral, CMUC & Universidade de Coimbra

The well known equivalence between group actions and split extensions can be extended to an equivalence between internal actions and split extensions in categories where the notion of semidirect product can be defined intrinsically. This equivalence holds in quite general contexts that include, for instance, all pointed varieties satisfying the split short five lemma, which is not the case of monoids. There we have, on one hand, (classical) monoid actions/semidirect products and, on the other hand, internal actions/ categorical semidirect products and they are not equivalent constructions, in general. In this seminar we show that monoid actions correspond to a certain family of split epimorphisms that we call Schreier split epimorphisms. Moreover, the category of Schreier split epimorphims in monoids shares many of the algebraic properties of the category of split epimorphisms in groups. In addition, by means of these split epimorphisms, we characterize groups amongst the monoids and, for any group in the category of monoids, we recover the equivalence between the categories of monoid actions, split extensions and internal actions.

Representation theories for lattices

Maria João Gouveia, CAUL & Universidade de Lisboa

There exist several representation theories for lattices:

- The well known Priestley duality for bounded distributive lattices which is natural in the sense of B. A. Davey and H. Werner.
- Urquharts topological representation for bounded lattices, making use of maximal disjoint filter-ideal pairs, extends Priestleys representation theory and establishes a bijective correspondence between bounded lattices and a special class of Stone spaces.
- Allwein and Hartonas full duality improves Urquharts representation for bounded lattices.
- Ploscicas representation theory for bounded lattices generalizes Priestley duality and uses the concept of a maximal partial homomorphism.

Every bounded lattice admits two unital semilattice reducts. This makes possible to establish a topological representation for bounded lattices making use of Hofmann-Mislove-Stralka natural duality for unital semilattices.

How do all these representations relate? And how different are they?

Descrições equacionais de linguagens reconhecíveis e aplicações

Mário Branco, CAUL & Universidade de Lisboa

Uma linguagem reconhecível sobre um conjunto (alfabeto) X é um subconjunto do monoide livre sobre X que é reconhecível por um autómato finito. Esta definição é equivalente a uma definição em termos de monoides finitos e ainda a uma definição em termos de congruências sobre o monoide livre. Estas caracterizações algébricas têm-se mostrado essenciais para estudar propriedades combinatórias de linguagens reconhecíveis. Certas classes importantes destas linguagens são caracterizadas por classes de monoides finitos, ditas pseudovariedades de monoides (Eilenberg e Schutzenberger), análogas às variedades de álgebras de Birkhoff. Algumas pseudovariedades são caracterizadas por identidades no sentido de Birkhoff, mas em geral as pseudovariedades são caracterizadas por identidades u=v, onde u e v são elementos de um completamento um espaço topológico construído a partir do monoide livre, ditas pseudoidentidades (Reiterman). Este completamento é um espaço de Stone. Almeida e Pippenger estabeleceram conexões fortes entre as linguagens reconhecíveis e os abertos fechados daquele completamento topológico. Almeida, implicitamente, e Pippenger, explicitamente, mostraram que a álgebra de Boole das linguagens reconhecíveis sobre X é dual do referido espaço de Stone. Em 2008, Gehrke, Grigorieff e Pin usaram os resultados de Pippenger e a dualidade de Priestley para caracterizarem certos tipos de subreticulados das linguagens reconhecíveis, através de de um novo conceito de identidades. Nestes seminários será apresentada uma visão geral destes resultados e, posteriormente, algumas aplicações.

3 Semigroups of transformations

Classification of the S_n-pairs

Jorge André, Universidade Nova de Lisboa & CAUL

In semigroup theory one of the major goals is certainly to see how the group of units shapes the structure of the semigroup. This topic has been attracting the attention of many mathematicians such as J. Arajo, W. Bentz, P. Cameron, J. Fountain, A. Garro, I. Levi, D. McAlister, R. McFadden, P. Medeiros, J. Mitchell, M. Neuhoffer, P. Neumann, J. Saxl, C. Schneider, B. Steinberg and many other. The general goal is to classify the pairs (a, G), where a is a non-invertible transformation and G is a group of permutations, such that the semigroup $< G, a >$ has a given property P, for example is regular or idempotent generated.

Let (a, G) be a pair as above. We say that (a, G) is an S_n-pair if

$$< G, a > \backslash G =< S_n, a > \backslash S_n.$$

The importance of classifying the S_n-pairs comes from the fact that almost everything is known about the semigroups $< S_n, a > \backslash S_n$. Therefore, the classification of the S_n-pairs gives for free, so to speak, almost everything about the semigroup $< G, a >$. In this talk we are going to give a characterization of the S_n-pairs and show how that was used to find an interesting new class of permutation groups.

This is a joint work with João Araújo and Peter J. Cameron.

On the maximal subsemigroups of some transformation semigroups

Ilinka Dimitrova, South-West University "Neofit Rilski"

For $n \in \mathbb{N}$, let $X_n = \{1 < 2 << n\}$ be a finite chain with n elements. As usual, we denote by PTn the semigroup of all partial transformations of X_n. We consider some subsemigroups of the semigroup PTn and characterize their maximal subsemigroups.

Semigroups of endomorphisms of a chain with restricted range

Vítor Hugo Fernandes, Universidade Nova de Lisboa & CAUL

Let X be a finite or infinite chain and let $O(X)$ be the monoid of all endomorphisms of X. In this talk, we describe the largest regular subsemigroup of $O(X)$ and Green's relations on $O(X)$. More generally, if Y a nonempty subset of X and $O(X, Y)$ is the subsemigroup of $O(X)$ of all elements with range contained in Y, we characterize the largest regular subsemigroup of $O(X, Y)$ and Green's relations on $O(X, Y)$. For finite chains, we also determine when two semigroups of the type $O(X, Y)$ are isomorphic and calculate their ranks.

Maximal semigroups satisfying $x = x^k$ in the ideal $K(n, 2)$ of T_n

Jörg Koppitz, Postdam University

We consider peroidic semigroups satisfying $x = x^k$. in the ideal $K(n, 2)$ of all transformation on an n-element set with an image size less or equal two. We can show that there is a one-to-one correspondence to equivalence relations. In fact, we characterize the maximal semigroups consisting only of involutions as well as all bands in $K(n, 2)$ by equivalence relations. We present an algorithm to construct such semigroups.