Surface Physics ain't Superficial:
A New Model of Wetting

Nelson R. Bernardino and Andrew O. Parry
Department of Mathematics, Imperial College London
nelson.bernardino@imperial.ac.uk
a.o.parry@imperial.ac.uk

Imperial College London

1. Abstract

Despite many successes, the "*standard model*" of wetting transitions has serious flaws. The origin of these flaws can be traced to the presence of nonlocal terms in the Hamiltonian describing the statistical mechanics of interfacial fluctuations.

We present the main results of the *nonlocal model* of wetting. These can be elegantly represented in a *diagrammatic form* (similar to Feynman diagrams). It is this representation that we can see in the *background* of the poster. The nonlocal model opens new doors for studying wetting at micro-patterned and structured substrates, a subject of enormous technological importance for *microfluidics* (fluids adsorbed at the microscale).

2. What is Wetting?

A liquid in contact with a solid substrate forms a droplet with a *contact angle*, θ (see figures). If $\theta = 0$ the droplet spreads and the substrate is covered by a film of liquid (the substrate is wet).

Wetting is a *phase transition* [3] at which the contact angle vanishes at a temperature T_{w}. Thus $\theta(T) = 0$ for $T > T_{\mathrm{w}}$. Equivalently, the mean thickness of the liquid layer, l becomes macroscopic, i.e. diverges to infinity.

The transition can be first-order or continuous (*critical wetting*). The latter is of great theoretical interest and is characterized by *critical exponents* describing the assymptotic behaviour of the thickness, l and the correlation length, $\xi_{\|}$, close to T_{w}:

$$l \sim (T_{\mathrm{w}} - T)^{-\beta}$$
$$\xi_{\|} \sim (T_{\mathrm{w}} - T)^{-\nu_{\|}}$$

3. The Standard Model

A simple model underlies the whole theory of wetting. The interface is considered to be a *surface of tension* Σ, described by the Hamiltonian (Energy)

$$H_{\mathrm{I}} = \int \mathrm{d}\vec{x} \left\{ \frac{\Sigma}{2} (\nabla l(\vec{x}))^2 + W(l) \right\}$$

$W(l)$ describes the interaction of the interface with the substrate:

$$W(l) = -a(T_{\mathrm{w}} - T)\mathrm{e}^{-\kappa l} + b\mathrm{e}^{-2\kappa l}$$

with a, b and κ positive constants.

4. Problems, Controversies, Disasters

✔ The Standard Model is correct for dimension $d \neq 3$.
✘ At $d = 3$ the model predicts $\nu_{\|} \approx 5$. In "experiments" $\nu_{\|} \approx 1$!
✘ Fails for non-planar substrates, e.g. a sphere or a wedge.
✘ Doesn't quite satisfy certain exact requirements (sum-rules).

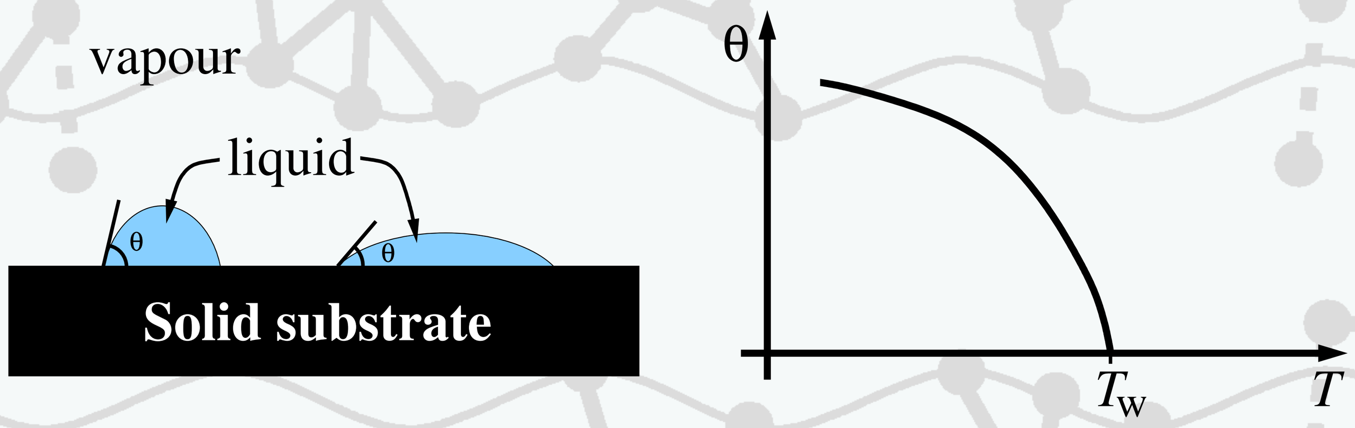


Figure 1: The contact angle, θ, is zero above the wetting temperature, T_{w}.

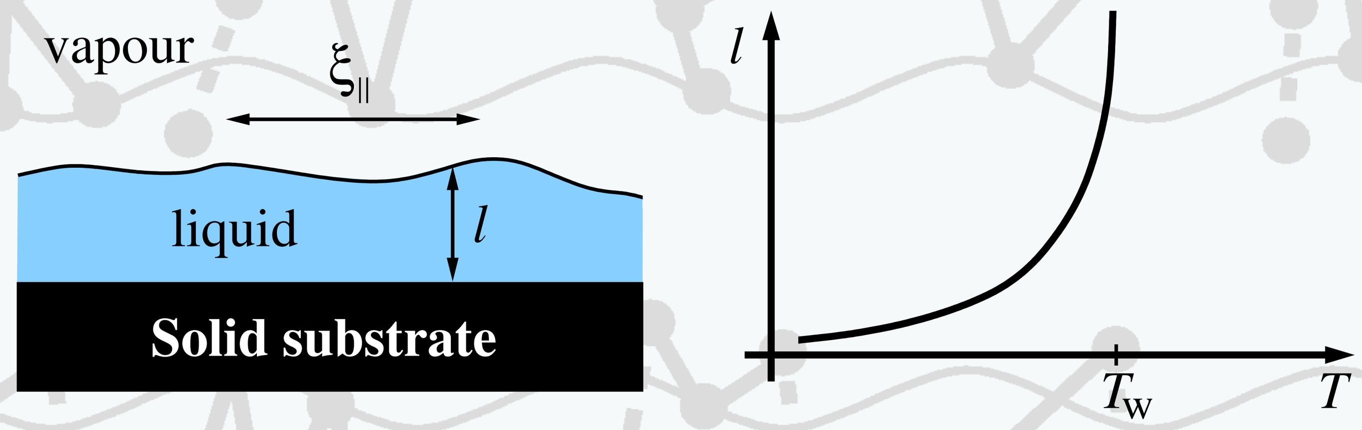
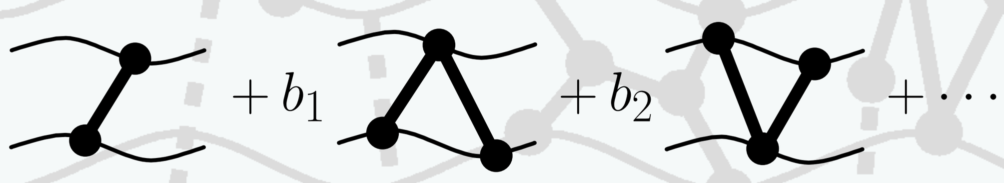


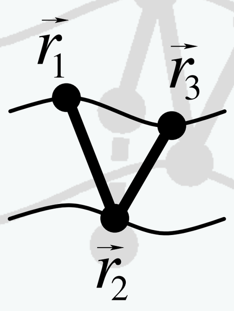
Figure 2: The thickness of a liquid film diverges to infinity at the wetting temperature, T_{w}.

5. The Nonlocal Model

Derivation of the Interfacial Model from a microscopic model reveals a *nonlocal interaction functional* [1,2]:

$W[l] = -a(T_{\mathrm{W}} - T)$ 

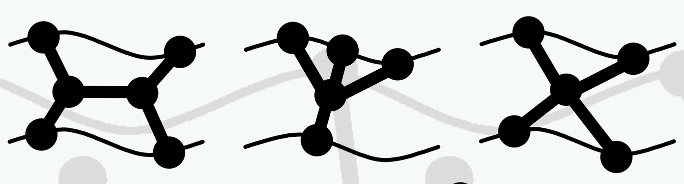
The diagrams represent integrations over the surfaces (black dots) with the kernel given by the Green's function of the Helmholtz equation (the straight line). Thus

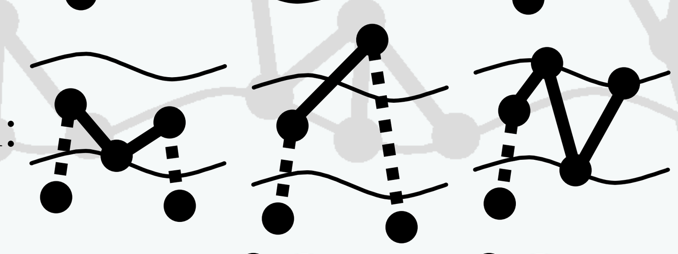


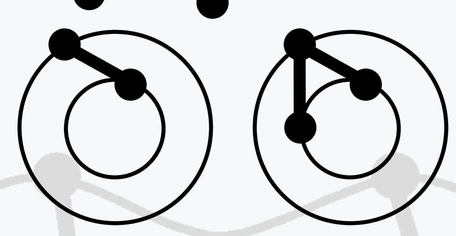
$$= \int \mathrm{d}\vec{S}_{\vec{r}_3} \int \mathrm{d}\vec{S}_{\vec{r}_2} K(\vec{r}_2, \vec{r}_3) \int \mathrm{d}\vec{S}_{\vec{r}_1} K(\vec{r}_1, \vec{r}_2)$$

6. Brave New World

✔ 3d wetting problem is solved.

✔ Can do perturbation theory: 

✔ Can introduce a surface potential: 

✔ Can do wetting on non-planar substrates: 

7. Take Home Message

- The simple model of wetting is not "good enough".
- A careful derivation reveals nonlocal interactions.
- The nonlocal model solves old problems in wetting.
- It allows us to do wetting in curved substrates.
- It has an elegant (beautiful!?) diagrammatic representation.

8. References & Acknowledgements

[1] A O Parry, C Rascón, N R Bernardino and J M Romero-Enrique, Journal of Physics: Condensed Matter, **18**, 6433 (2006).
[2] A O Parry, J M Romero-Enrique and A Lazarides, Physical Review Letters, **93**, 086104 (2004).
[3] M Schick, *Liquids at interfaces*, J Charvolin, J F Joanny and J Zinn-Justin (Eds), Elsevier (1990).

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