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# *Wetting in a Nutshell*

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Maths Postgraduate Seminar  
October, 26<sup>th</sup> 2006  
Imperial College London

- <http://www.ma.imperial.ac.uk/~nfbern/Docs/notes.pdf>
- A O Parry, C Rascon, N R Bernardino, J M Romero-Enrique, J. Phys.: Condens. Matter, **18**, 6433 (2006)

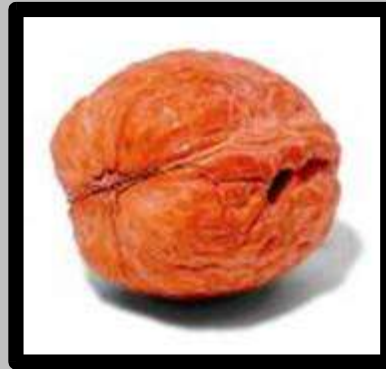
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# ***Outlook***

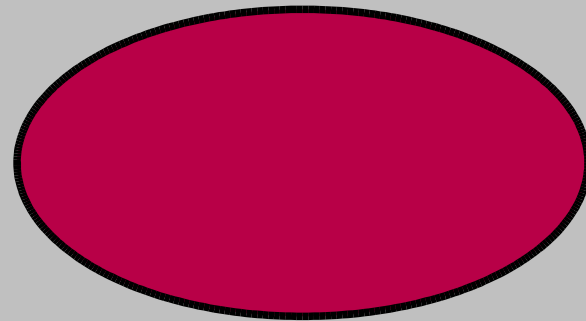
- Basic Concepts of Wetting
- Review of Critical Phenomena
- Interfacial Model (Ad Hoc Version)
- Interfacial Model Revisited (Systematic)
- New Results and Their Meaning

# *Nutshell*



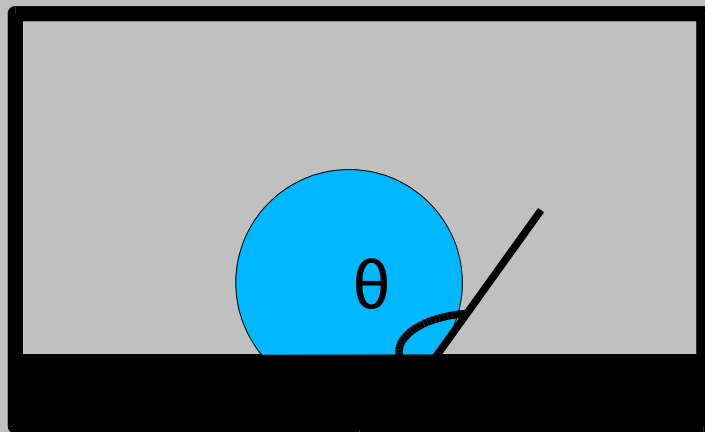
Mathematical Nutshell:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

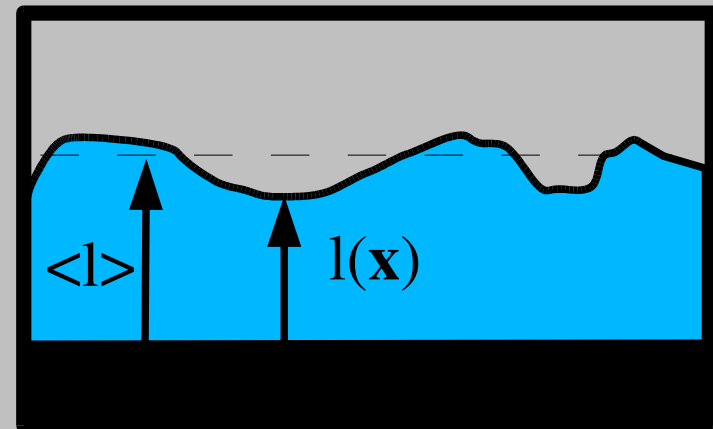


# Wetting

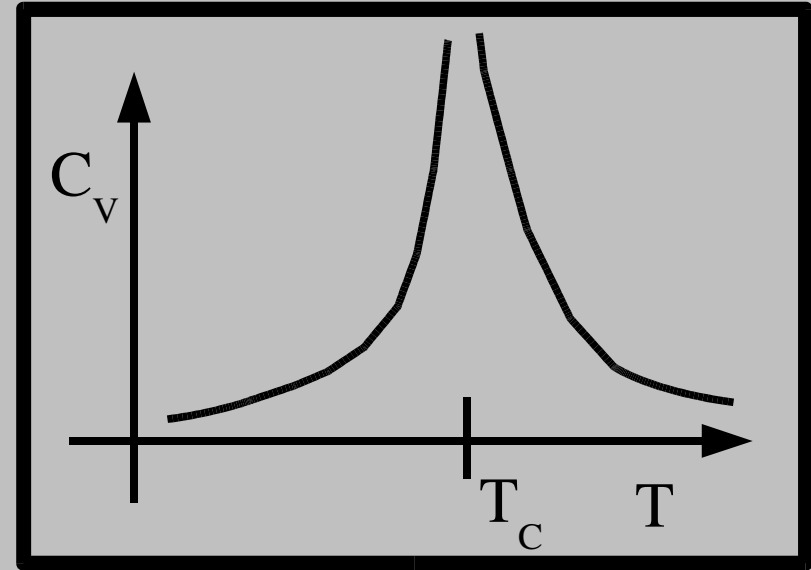
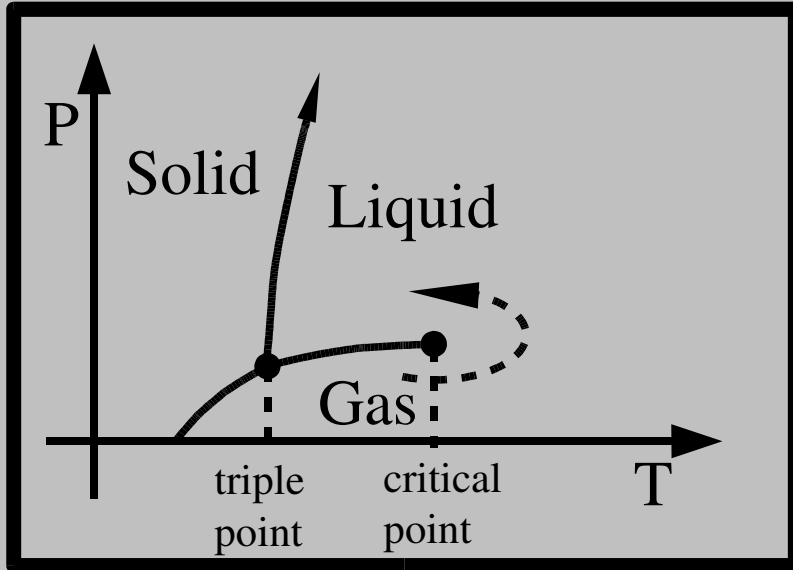
- Interfacial phase transition
- Contact angle goes to zero
- Mean interfacial height goes to infinity



or, with an infinite number of particles:



# Critical Phenomena



- Free-energy is non-analytic
- Fluctuations are HUGE!!!
- Power-law singularities described by a set of critical exponents, e.g.  $C_v \sim |T - T_c|^{-\alpha}$

# Landau-Ginzburg-Wilson Theory I

- Model:

$$H_{LGW} = \int d\mathbf{r} \left[ \frac{(\nabla m(\mathbf{r}))^2}{2} + \Phi(m(\mathbf{r})) \right]$$

$$\Phi(m) = -t \frac{m^2}{2} + u \frac{m^4}{4}$$

- Statistical Physics:

$$Z = \int D_m e^{-\beta H}$$

- Thermodynamics:

$$F = -k_B T \ln Z$$

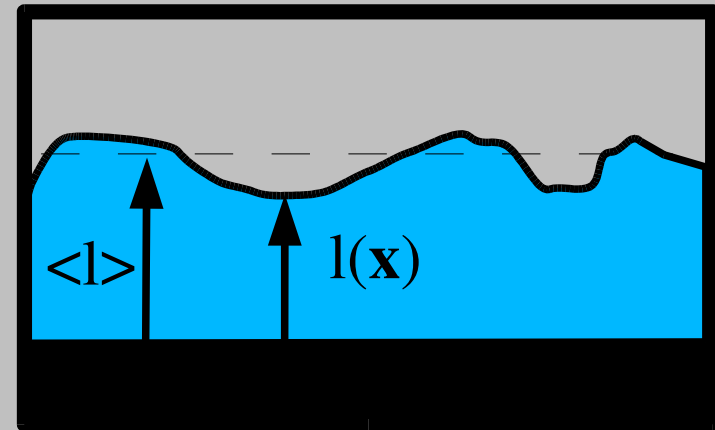
# *Landau-Ginzburg-Wilson Theory II*

- Mean-Field Theory (MF) = Minimize  $H_{LGW}$ 
  - Wrong critical exponents
  - Underestimates fluctuations
  - OK (usually) far from critical point
- Renormalization Group Theory (RG)
  - Accounts correctly for fluctuations
  - **Universality!!!**
  - Predicts MF is correct for  $d > d_c$   
(**upper critical dimension**)

# Interfacial Model

- LGW model is too difficult - translational invariance broken.
- Phenomenological theory: focus on interface, described as stretched membrane.

$$H_I = \int d\mathbf{x} \left[ \frac{\Sigma (\nabla l(\mathbf{x}))^2}{2} + W(l(\mathbf{x})) \right]$$



$$W(l) = -a e^{-kl} + b e^{-2kl}$$

# *Interfacial Model II*

- Upper Critical Dimension = 3
- RG predicts non-universal (detail dependent) results in *3D*

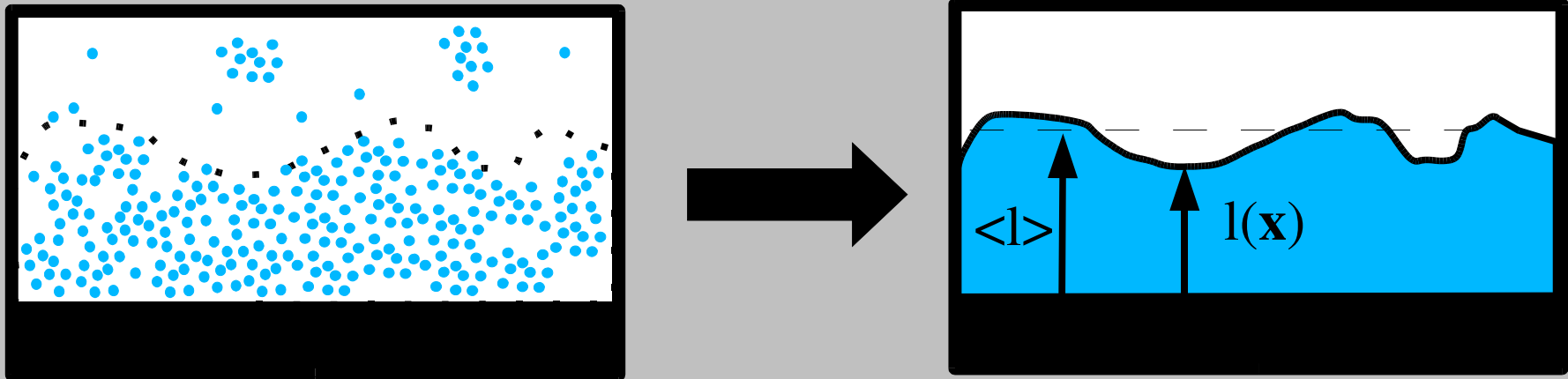
Simulations of Interfacial model agree with RG

Simulations of LGW model agree with MF

Something is **wrong** with the **details** of the interfacial model!!!

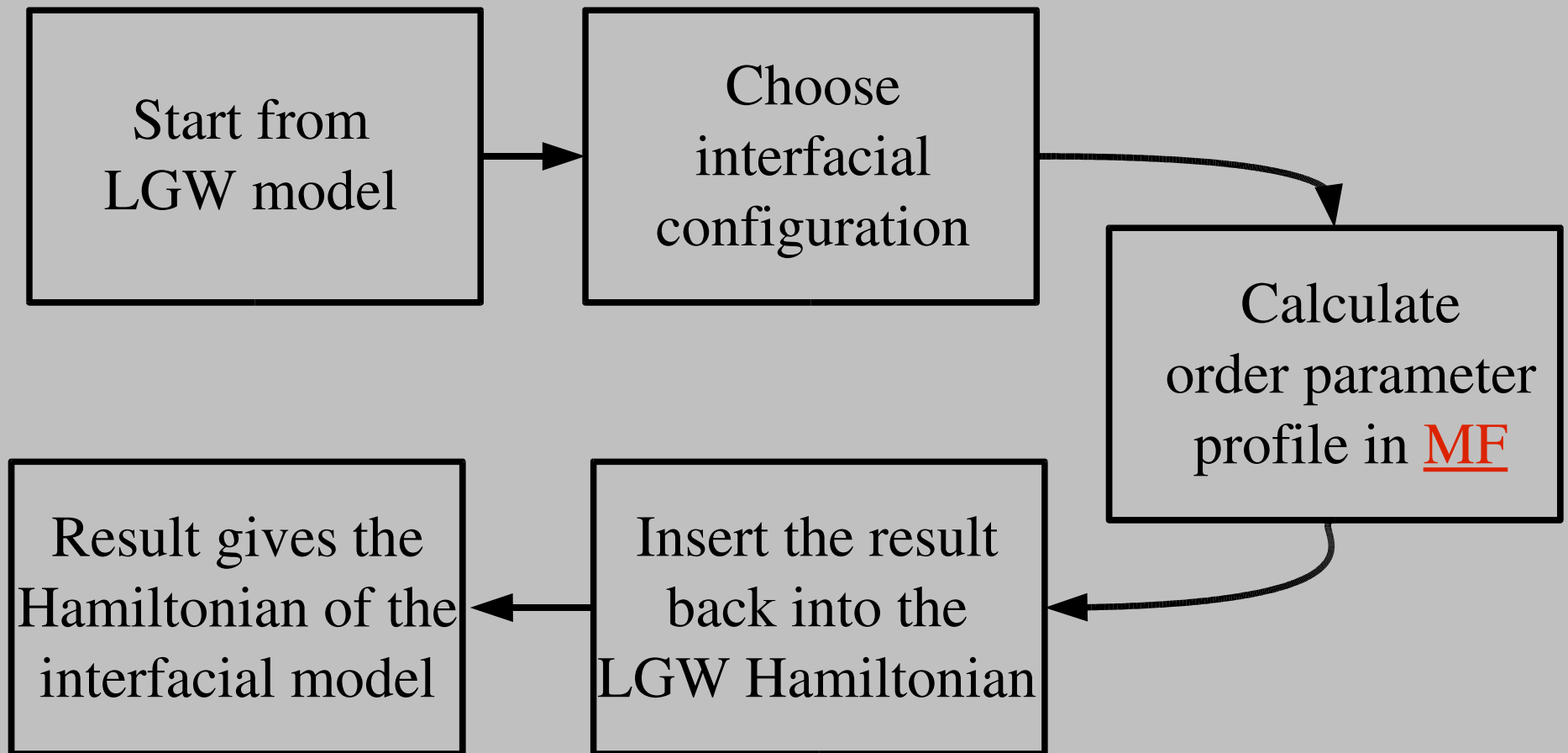
# *Interfacial Model Revisited*

- How to derive the interfacial model from the LGW model?



“Integrate” irrelevant details using MF.  
Interface defined as  $m=0$  curve.

# Rigorous Definition of the Interfacial Model I



# ***Rigorous Definition of the Interfacial Model II***

- Functional minimization of  $H_{\text{LGW}}$  with required interface in MF leads to:

$$\nabla^2 m_{\Xi} = k^2 (m_{\Xi} \pm m_0)$$

with boundary conditions:

$$m_{\Xi}(\infty) = -m_0 \quad m_{\Xi}(l) = 0 \quad m_{\Xi}(\Psi) = m_1$$

# Solving the Helmholtz Equation

$$m_{\equiv} = -m_0 + m_0 \text{ [diagram: a wavy line with a solid dot and a line to an open circle above it]} \quad ; \text{ above the interface}$$

$$m_{\equiv} = m_0$$

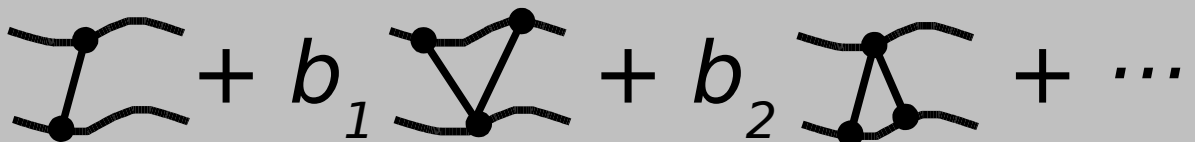
$$+ (m_1 - m_0) \text{ [diagram: two wavy lines with a solid dot on the top line and a line to an open circle on the bottom line]} + m_0 \text{ [diagram: two wavy lines with two solid dots on the top line and lines to two open circles on the bottom line]} + (m_1 - m_0) \text{ [diagram: two wavy lines with two solid dots on the top line and lines to two open circles on the bottom line]} + \dots$$

$$- m_0 \text{ [diagram: two wavy lines with a solid dot on the top line and a line to an open circle on the bottom line]} - (m_1 - m_0) \text{ [diagram: two wavy lines with two solid dots on the top line and lines to two open circles on the bottom line]} - m_0 \text{ [diagram: two wavy lines with two solid dots on the top line and lines to two open circles on the bottom line]} - \dots$$

; below the interface

# *The Result and its Meaning*

- Replacing back in the LGW Hamiltonian:

$$W[l, \Psi] = a \text{ (diagram 1) } + b_1 \text{ (diagram 2) } + b_2 \text{ (diagram 3) } + \dots$$


- **Nonlocal** (Important Physics)
- Obeys exact statistical mechanical results (sum rules)
- Can be interpreted as interactions due to tube-like fluctuations.

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## ***Brave New World!!!***

- Renormalization results are same as previously but critical region is too small – only MF will be observed.
- New Result: With an exponentially decaying substrate-fluid interaction we can “switch off” nonlocal effects – full non-universal results should be observed.
- Non-ambiguous framework to study wetting in non-planar substrates.

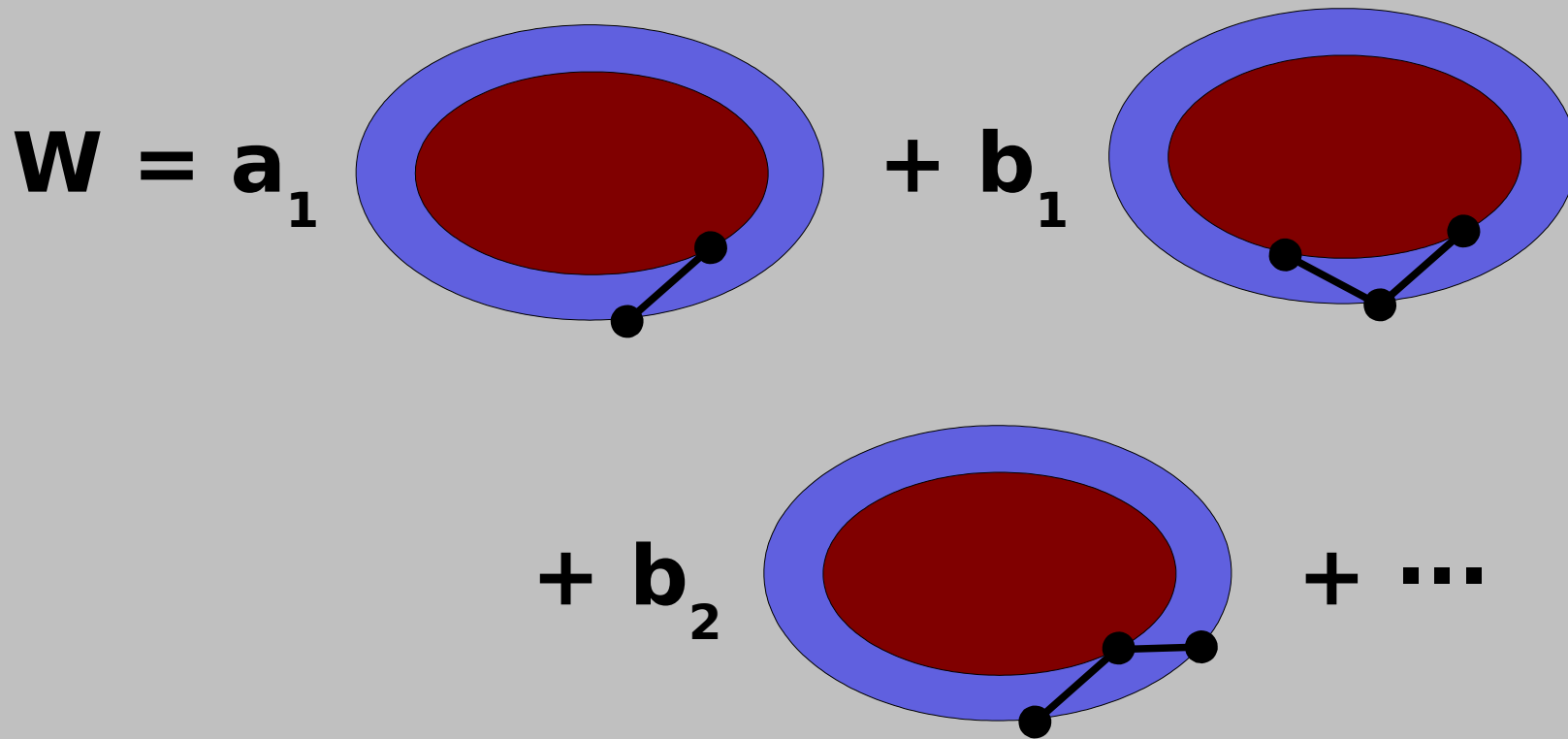
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## ***Take Home Message***

- **Usually** details of models are irrelevant for critical phenomena - **Universality**.
- At the **Upper Critical Dimension** things might not be universal.
- **Details** of the model are **important** for ***3D*** critical wetting.
- Careful derivation of interfacial model uncovers important physical effects: **nonlocal** model.
- The nonlocal model allows us to do **wetting in curved substrates**.

# *Wetting in a Nutshell*




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## ***Done With***

- Andy Parry (Imperial)
- Laura Morgan (Imperial)
- Carlos Rascon (Madrid)
- Jose Manuel Romero-Enrique (Sevilla)



"You do not really understand something unless you can explain it to your grandmother."

Albert Einstein



"You do not really understand something unless you can explain it to ~~your grandmother~~ maths postgrads."

Albert Einstein

Thank you for listening