Wetting of hairy surfaces

Nelson R. Bernardino & Siegfried Dietrich

Max-Planck-Institut für Metallforschung
Stuttgart, Germany

Lisboa, 17 June 2009
Wetting and Geometry

Geometry → Wetting
- Capillary condensation.
- Wedge wetting.
- Superhydrophobic surfaces.¹

Geometry ← Wetting
- Deformation of elastic surfaces.
- Collapse of patterned substrates.¹

Goal: Geometry ⇐ Wetting.

¹Lau et al, Nanoletters 3 (2003), 1701
An Intriguing Example: The Lady’s Mantle

- Leaves covered with hydrophilic hairs.¹
- Droplets can be suspended in the fur.

- Behaviour attributed to elasticity of the hairs.

Simple Model I: Elastic Energy

Elastic Energy

\[ E_{\text{el}} = \frac{3\pi ER^4}{8h^3} u^2 \]

- \( E \equiv \) Young’s modulus.
- \( R \equiv \) radius of posts.
- \( h \equiv \) height.
- \( u \equiv \) deviation from equilibrium position.
Simple Model II: Capillary Interaction

Capillary Energy

\[ E_{\text{cap}} = 2\pi \sigma R^2 \ln(qL) \cos^2 \theta \]

- \( \sigma \equiv \) surface tension.
- \( \theta \equiv \) contact angle.
- \( q^{-1} \equiv \) capillary length.
- \( L \equiv \) distance between centres of posts.
Wetting Energy

\[ E_{\text{wet}} = 2\pi Rh\sigma \cos\theta \]
Some Remarks

- Thermal fluctuations are too small.
- Non-dimensionalising we have

\[ \bar{H} = \bar{h} \cos \theta + \frac{\bar{k}}{\bar{h}^3} \bar{u} + \ln(\bar{L}) \cos^2 \theta \]

- Essential parameter:

\[ \bar{k} = \frac{3ER}{8\sigma} \]

- The Young’s Modulus of the Lady’s Mantle hairs is not known but a lower estimate is \( \bar{k} \gtrsim 50 \).
$H(u, h) = h \cos \theta + \frac{k}{h^3} u^2 + \frac{\ln(L - 2u) \cos^2 \theta}{2} + \frac{\ln(L + 2u) \cos^2 \theta}{2}$

$u \in \left[0, \frac{L - 2}{2}\right]$
$k = 1, \ L = 30, \ \cos \theta = 1.$
$k = 1, \quad L = 30, \quad \cos \theta = 1.$
\[ k = 1, \quad L = 30, \quad \cos \theta = 1. \]
$k = 1, L = 30, \cos \theta = 1.$
$k = 1, \ L = 30, \ \cos \theta = 1.$
Simple Estimates

- Posts bend if
  \[ h^3 > \frac{kL^2}{2 \cos^2 \theta} \]

- Elastic energy balances wetting energy when
  \[ h^*^4 = \frac{k(L - 2)^2(n^2 - 1)}{4 \cos \theta} \]

- Cluster holds together if
  \[ (L - 2)^2 > 2^{10} k \frac{(n - 1)^4}{(n^2 - 1)^3 (\cos \theta)^5} \]
Simple Estimates

- Posts bend if
  \[ h^3 > \frac{kL^2}{2 \cos^2 \theta} \]

- Elastic energy balances wetting energy when
  \[ h^4 = \frac{k(L - 2)^2(n^2 - 1)}{4 \cos \theta} \]

- Cluster holds together if
  \[ (L - 2)^2 > 2^{10} k \frac{(n - 1)^4}{(n^2 - 1)^3 \cos^5 \theta} \]
Simple Estimates

- Posts bend if
  \[ h^3 > \frac{kL^2}{2\cos^2\theta} \]

- Elastic energy balances wetting energy when
  \[ h^*4 = \frac{k(L - 2)^2(n^2 - 1)}{4\cos\theta} \]

- Cluster holds together if
  \[ (L - 2)^2 > 2^{10}k\frac{(n - 1)^4}{(n^2 - 1)^3\cos^5\theta} \]
\( (k = 1, \cos \theta = 1) \Rightarrow \quad L \gtrsim 8, h^* \approx 5. \)

Lady’s Mantle: \( (k = 50, \cos \theta = 1/2) \Rightarrow \quad L \gtrsim 250, h^* \approx 50! \)

With bigger clusters and pair interactions \( L \) can be smaller.

1D \( \neq \) 2D: For Lady’s Mantle

\( L \gtrsim 90, h^* \approx 30. \)
\( (k = 1, \cos \theta = 1) \Rightarrow \]
\[
L \gtrsim 8, h^* \approx 5.
\]

\( \text{Lady’s Mantle: } (k = 50, \cos \theta = 1/2) \Rightarrow \]
\[
L \gtrsim 250, h^* \approx 50!.
\]

- With bigger clusters and pair interactions \( L \) can be smaller.
- 1D \( \neq \) 2D: For Lady’s Mantle

\[
L \gtrsim 90, h^* \approx 30.
\]
$(k = 1, \cos \theta = 1) \Rightarrow \quad L \gtrsim 8, \, h^* \approx 5.$

Lady’s Mantle: $(k = 50, \cos \theta = 1/2) \Rightarrow \quad L \gtrsim 250, \, h^* \approx 50!$

With bigger clusters and pair interactions $L$ can be smaller.

$1D \neq 2D$: For Lady’s Mantle

$L \gtrsim 90, \, h^* \approx 30.$
Simple Estimates II

- \((k = 1, \cos \theta = 1) \Rightarrow \)
  \[ L \gtrsim 8, h^* \approx 5. \]

- Lady’s Mantle: \((k = 50, \cos \theta = 1/2) \Rightarrow \)
  \[ L \gtrsim 250, h^* \approx 50! \]

- With bigger clusters and pair interactions \(L\) can be smaller.

- 1D \(\neq\) 2D: For Lady’s Mantle
  \[ L \gtrsim 90, h^* \approx 30. \]
Back to the Lady’s Mantle

Thanks to Valentin Blickle (and mom)
Thanks to Valentin Blickle (and mom)
Back to the Lady’s Mantle

Thanks to Valentin Blickle (and mom)
Back to the Lady’s Mantle

Thanks to Valentin Blickle (and mom)
Thanks to Valentin Blickle (and mom)
Back to the Lady’s Mantle

Thanks to Valentin Blickle (and mom)
Back to the Lady’s Mantle

Thanks to Valentin Blickle (and mom)
Back to the Lady’s Mantle

Thanks to Valentin Blickle (and mom)
Accurate estimate of energy of collapsed posts.
Many body capillary interactions.
Influence of lattice/disorder.
Influence of inclined posts.
Curvature of interface.
Experimental verification of contact angle and hairs’ Young’s modulus.