

# Wetting of hairy surfaces

Nelson Rei Bernardino<sup>1</sup>, Valentin Blickle<sup>2</sup> & Siegfried Dietrich<sup>1</sup>

<sup>1</sup>Max-Planck-Institut für Metallforschung  
Stuttgart, Germany

<sup>2</sup>Physikalisches Institut  
Universität Stuttgart, Germany

Brookhaven National Laboratory  
27 July 2009

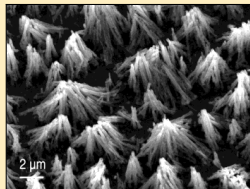
## Geometry $\rightarrow$ Wetting

- Capillary condensation.
- Wedge wetting.
- Superhydrophobic surfaces.<sup>1</sup>



## Geometry $\leftarrow$ Wetting

- Deformation of elastic surfaces.
- Collapse of patterned substrates.<sup>1</sup>

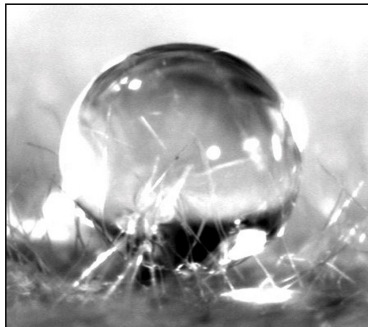


Goal: Geometry  $\Leftrightarrow$  Wetting.

<sup>1</sup>Lau et al, Nanoletters **3** (2003), 1701

# An Intriguing Example: The Lady's Mantle

- Droplets can be suspended in the fur:



- Hairs are hydrophilic.<sup>1</sup>
- Behaviour attributed to elasticity of the hairs.

---

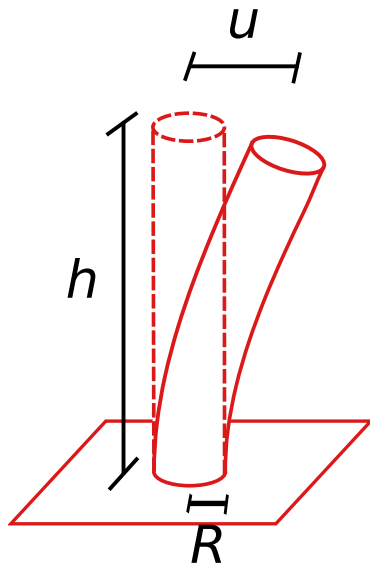
<sup>1</sup>Otten & Herminghaus, *Langmuir* **20** (2004) 2405

# Simple Model I: Elastic Energy

- Elastic Energy

$$E_{\text{el}} = \frac{3\pi ER^4}{8h^3} u^2$$

- $E \equiv$  Young's modulus.
- $R \equiv$  radius of posts.
- $h \equiv$  height.
- $u \equiv$  deviation from equilibrium position.

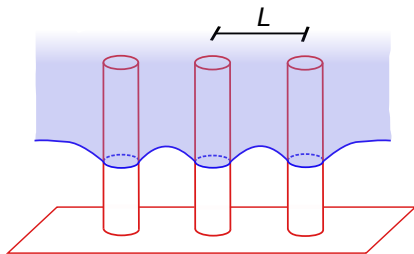


# Simple Model II: Capillary Interaction

- Capillary Energy

$$E_{\text{cap}} = 2\pi\sigma R^2 \ln(qL) \cos^2 \theta$$

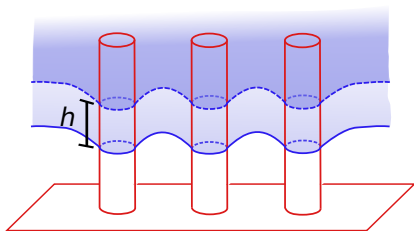
- $\sigma \equiv$  surface tension.
- $\theta \equiv$  contact angle.
- $q^{-1} \equiv$  capillary length.
- $L \equiv$  distance between centres of posts.



# Simple Model III: Wetting Energy

- Wetting Energy

$$E_{\text{wet}} = 2\pi Rh\sigma \cos\theta$$



# Some Remarks

- Thermal fluctuations are too small.
- Non-dimensionalising we have

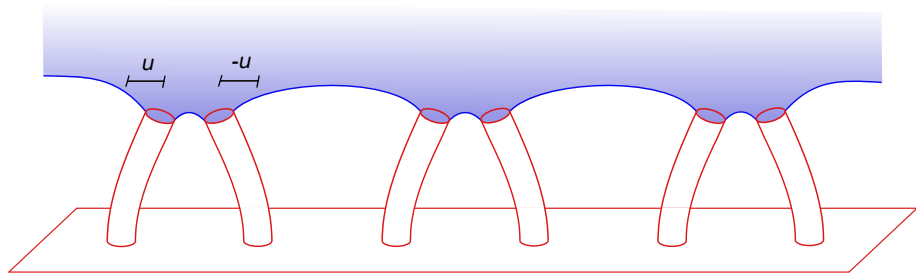
$$\bar{H} = \bar{h} \cos \theta + \frac{\bar{k}}{\bar{h}^3} \bar{u} + \ln(\bar{L}) \cos^2 \theta$$

- Essential parameter:

$$\bar{k} = \frac{3ER}{8\sigma}$$

- The Young's Modulus of the Lady's Mantle hairs is not known but a lower estimate is  $\bar{k} \gtrsim 50$ .

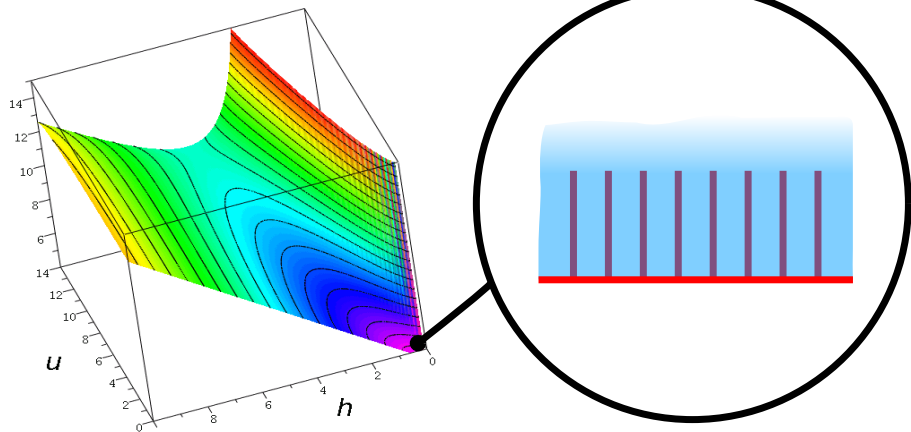
# 1D Toy Model I



$$H(u, h) = h \cos \theta + \frac{k}{h^3} u^2 + \frac{\ln(L - 2u) \cos^2 \theta}{2} + \frac{\ln(L + 2u) \cos^2 \theta}{2}$$

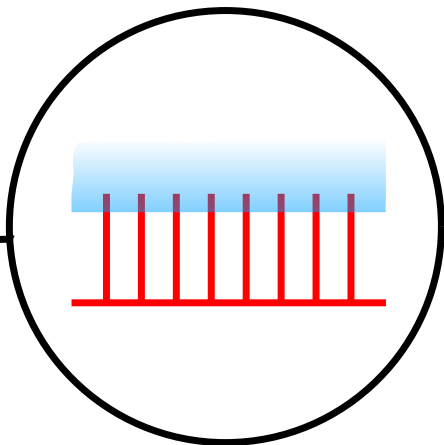
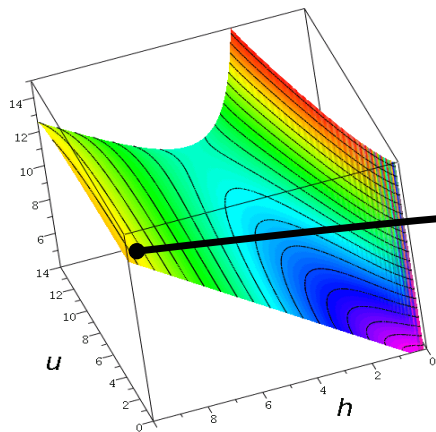
$$u \in \left[ 0, \frac{L - 2}{2} \right]$$

# 1D Toy Model II



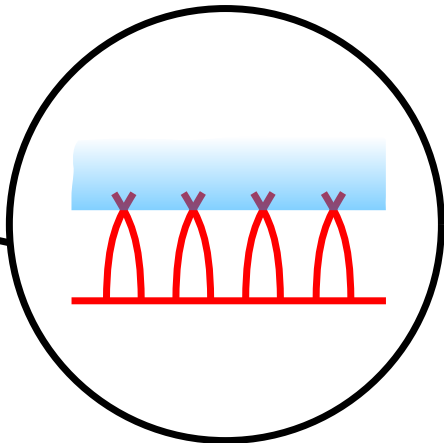
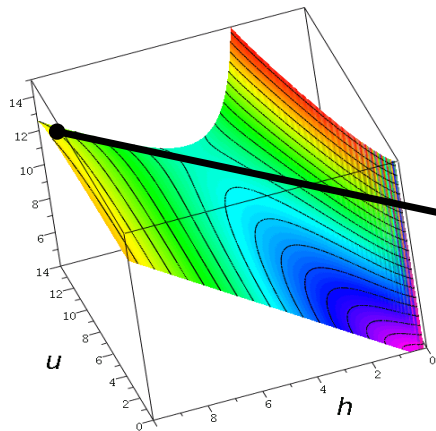
$$k = 1, L = 30, \cos \theta = 1.$$

# 1D Toy Model II



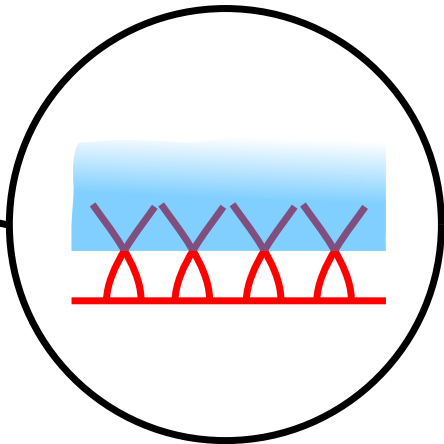
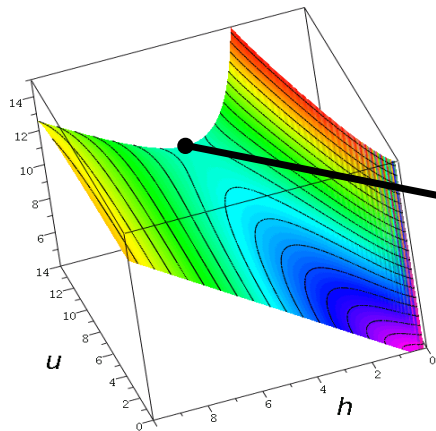
$$k = 1, L = 30, \cos \theta = 1.$$

# 1D Toy Model II



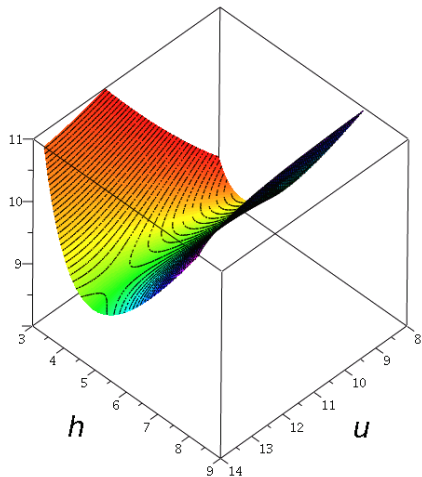
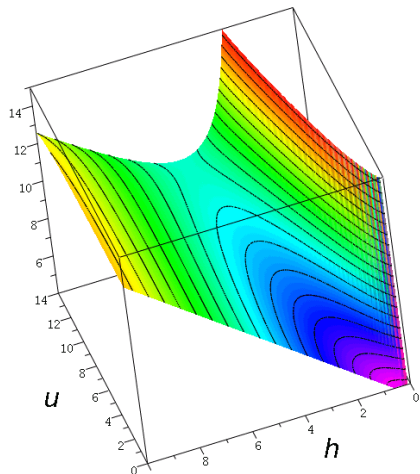
$$k = 1, L = 30, \cos \theta = 1.$$

# 1D Toy Model II



$$k = 1, L = 30, \cos \theta = 1.$$

# 1D Toy Model II



$k = 1, L = 30, \cos \theta = 1.$

# Simple Estimates

- Posts bend if

$$h^3 > \frac{kL^2}{2 \cos^2 \theta}$$

- Elastic energy balances wetting energy when

$$h^{*4} = \frac{k(L-2)^2(n^2-1)}{4 \cos \theta}$$

- Cluster holds together if

$$(L-2)^2 > 2^{10} k \frac{(n-1)^4}{(n^2-1)^3 (\cos \theta)^5}$$

# Simple Estimates

- Posts bend if

$$h^3 > \frac{kL^2}{2 \cos^2 \theta}$$

- Elastic energy balances wetting energy when

$$h^{*4} = \frac{k(L-2)^2(n^2-1)}{4 \cos \theta}$$

- Cluster holds together if

$$(L-2)^2 > 2^{10} k \frac{(n-1)^4}{(n^2-1)^3 (\cos \theta)^5}$$

- Posts bend if

$$h^3 > \frac{kL^2}{2 \cos^2 \theta}$$

- Elastic energy balances wetting energy when

$$h^{*4} = \frac{k(L-2)^2(n^2-1)}{4 \cos \theta}$$

- Cluster holds together if

$$(L-2)^2 > 2^{10} k \frac{(n-1)^4}{(n^2-1)^3 (\cos \theta)^5}$$

# Simple Estimates II

- $(k = 1, \cos \theta = 1) \Rightarrow$

$$L \gtrsim 8, h^* \simeq 5.$$

- Lady's Mantle:  $(k = 50, \cos \theta = 1/2) \Rightarrow$

$$L \gtrsim 250, h^* \simeq 50!$$

- With bigger clusters and pair interactions  $L$  can be smaller.
- $1D \neq 2D$ : For Lady's Mantle

$$L \gtrsim 90, h^* \simeq 30.$$

# Simple Estimates II

- $(k = 1, \cos \theta = 1) \Rightarrow$

$$L \gtrsim 8, h^* \simeq 5.$$

- Lady's Mantle:  $(k = 50, \cos \theta = 1/2) \Rightarrow$

$$L \gtrsim 250, h^* \simeq 50!$$

- With bigger clusters and pair interactions  $L$  can be smaller.
- 1D  $\neq$  2D: For Lady's Mantle

$$L \gtrsim 90, h^* \simeq 30.$$

# Simple Estimates II

- $(k = 1, \cos \theta = 1) \Rightarrow$

$$L \gtrsim 8, h^* \simeq 5.$$

- Lady's Mantle:  $(k = 50, \cos \theta = 1/2) \Rightarrow$

$$L \gtrsim 250, h^* \simeq 50!$$

- With bigger clusters and pair interactions  $L$  can be smaller.
- 1D  $\neq$  2D: For Lady's Mantle

$$L \gtrsim 90, h^* \simeq 30.$$

# Simple Estimates II

- $(k = 1, \cos \theta = 1) \Rightarrow$

$$L \gtrsim 8, h^* \simeq 5.$$

- Lady's Mantle:  $(k = 50, \cos \theta = 1/2) \Rightarrow$

$$L \gtrsim 250, h^* \simeq 50!$$

- With bigger clusters and pair interactions  $L$  can be smaller.
- $1D \neq 2D$ : For Lady's Mantle

$$L \gtrsim 90, h^* \simeq 30.$$

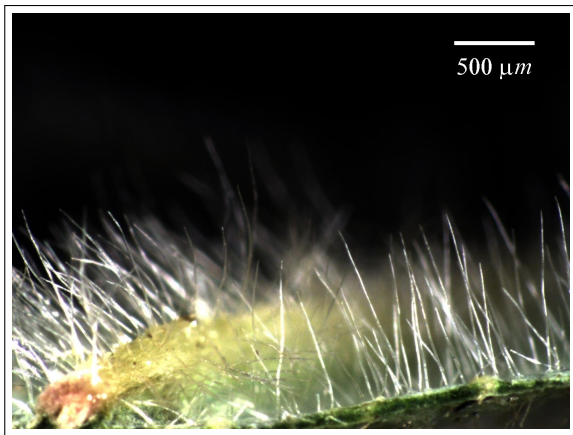
# Help from Valentin Blickle



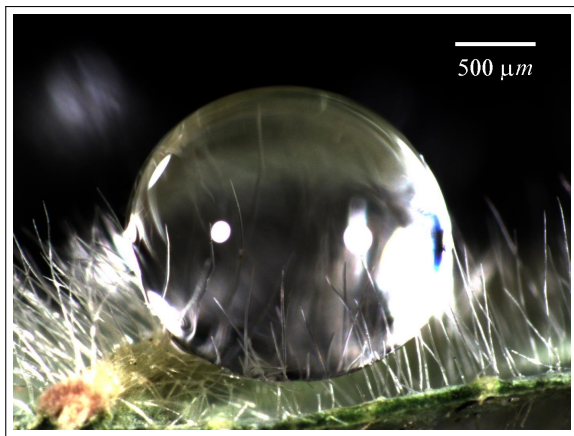
# Help from Valentin Blickle



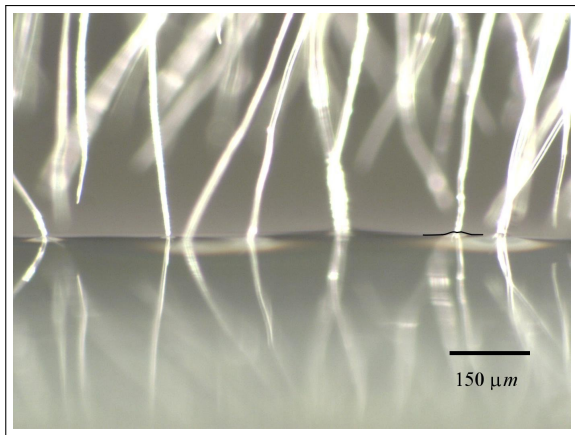
# Back to the Lady's Mantle



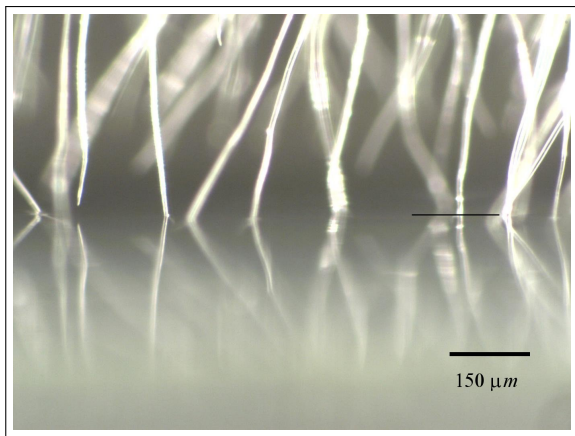
# Back to the Lady's Mantle



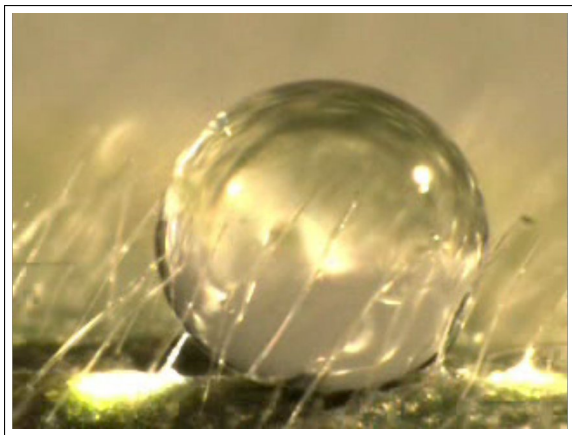
# Back to the Lady's Mantle



# Back to the Lady's Mantle



# Back to the Lady's Mantle



# Back to the Lady's Mantle

