

Wetting of hairy surfaces v2.0

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Wetting and Substrate Geometry

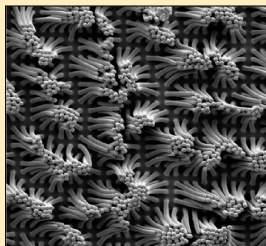
Geometry \rightarrow Wetting

- Capillary condensation.
- Wedge wetting.
- Superhydrophobic surfaces.¹



Geometry \leftarrow Wetting

- Deformation of elastic surfaces.
- Collapse of patterned substrates.²



Goal: Geometry \Leftrightarrow Wetting.

¹Lau *et al*, *Nanoletters* **3** (2003), 1701

²Chandra *et al*, *ACS Appl. Mater. Interfaces* **1** (2009), 1968

An Intriguing Example: The Lady's Mantle

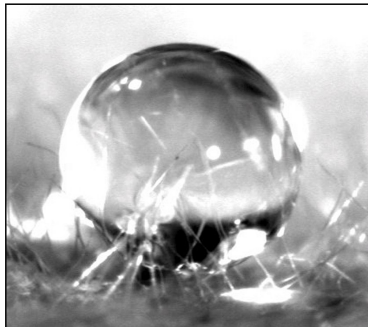


An Intriguing Example: The Lady's Mantle



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- Droplets can be suspended in the fur:



- Hairs are hydrophilic.¹
- Behavior attributed to elasticity of the hairs.

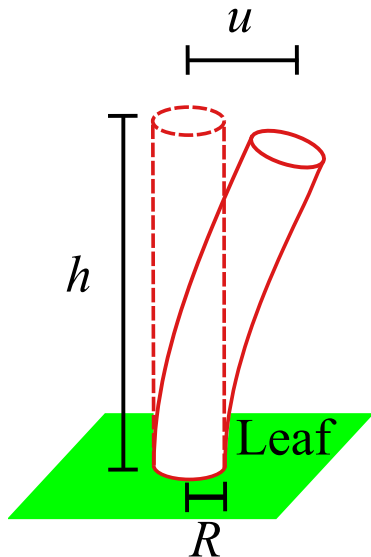
¹Otten & Herminghaus, *Langmuir* **20** (2004) 2405

Simple Model I: Elastic Energy

- Elastic Energy

$$E_{el} = \frac{3\pi ER^4}{8h^3} u^2$$

- $E \equiv$ Young's modulus.
- $R \equiv$ radius of posts.
- $h \equiv$ height.
- $u \equiv$ deviation from equilibrium position.

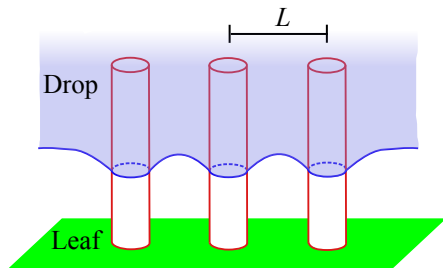


Simple Model II: Capillary Interaction

- Capillary Energy

$$E_{\text{cap}} = 2\pi\sigma R^2 \ln(qL) \cos^2 \theta$$

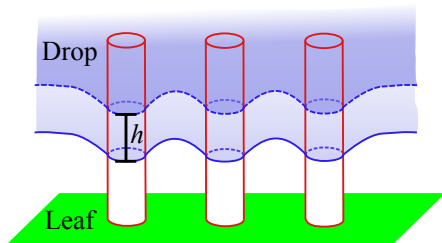
- $\sigma \equiv$ surface tension.
- $\theta \equiv$ contact angle.
- $q^{-1} \equiv$ capillary length.
- $L \equiv$ distance between centers of posts.



Simple Model III: Wetting Energy

- Wetting Energy

$$E_{\text{wet}} = 2\pi R h \sigma \cos \theta$$



Some Remarks

- Thermal fluctuations are too small.
- Non-dimensionalising we have

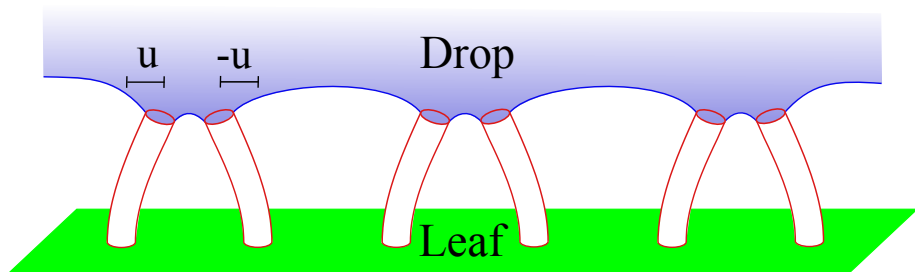
$$\bar{H} = \bar{h} \cos \theta + \frac{\bar{k}}{\bar{h}^3} \bar{u} + \ln(\bar{L}) \cos^2 \theta$$

- Essential parameter:

$$\bar{k} = \frac{3ER}{16\sigma}$$

- The Young's Modulus of the Lady's Mantle hairs is not known but a lower estimate is $\bar{k} \gtrsim 50$.

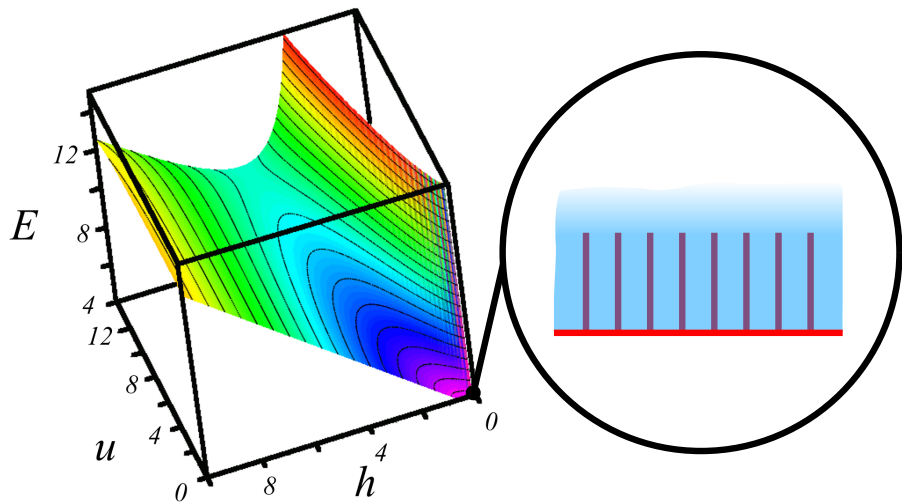
1D Toy Model I



$$H(u, h) = h \cos \theta + \frac{k}{h^3} u^2 + \frac{\ln(L - 2u) \cos^2 \theta}{2} + \frac{\ln(L + 2u) \cos^2 \theta}{2}$$

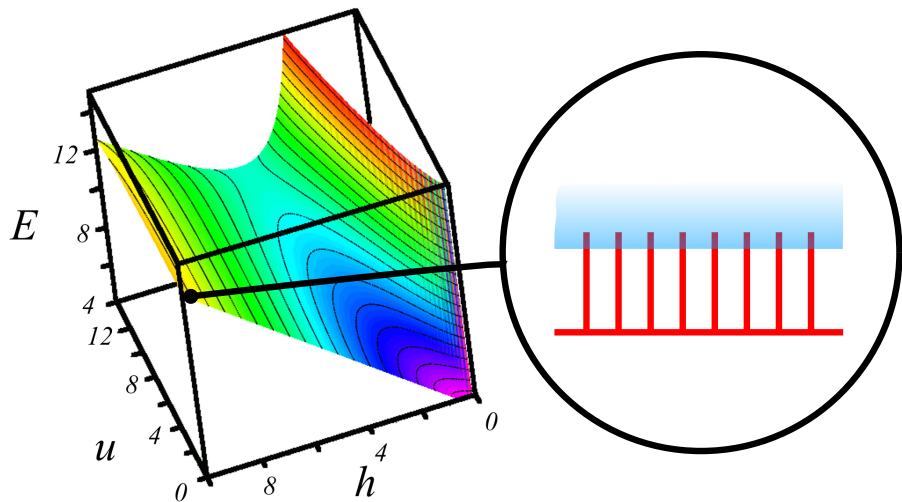
$$u \in \left[0, \frac{L - 2}{2} \right]$$

1D Toy Model II



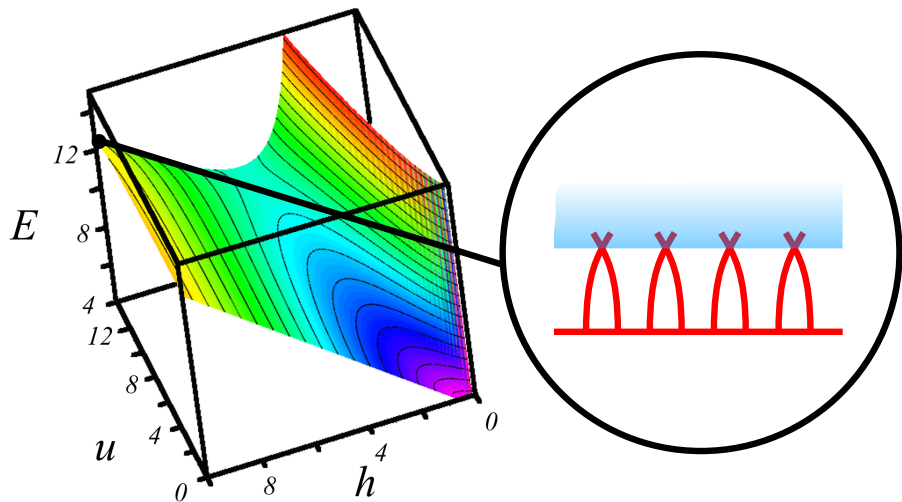
$$k = 1, L = 30, \cos \theta = 1.$$

1D Toy Model II



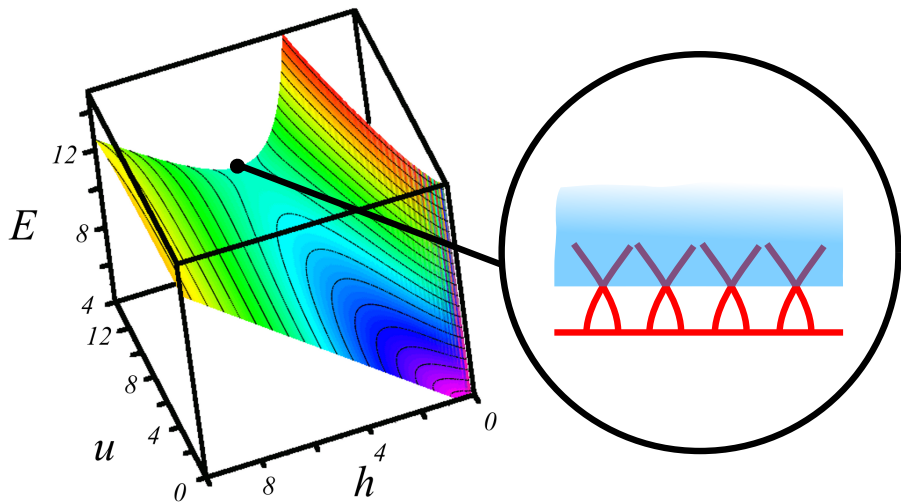
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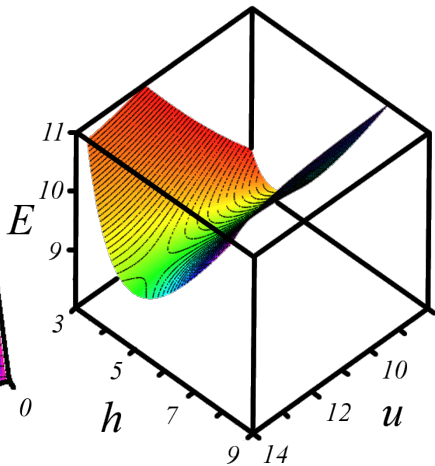
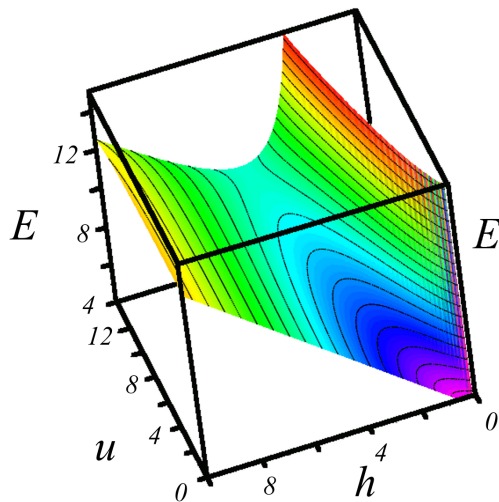
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Simple Estimates

- Posts bend if

$$h^3 > \frac{kL^2}{2 \cos^2 \theta}$$

- Elastic energy balances wetting energy when

$$h^{*4} = \frac{3k(L-2)^2}{4 \cos \theta}$$

- Cluster holds together if

$$L > 2 + 32 \sqrt{\frac{k}{27 \cos^5 \theta}}$$

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Simple Estimates II

- $(k = 1, \cos \theta = 1) \Rightarrow$

$$L \gtrsim 8, h^* \simeq 5.$$

- Lady's Mantle: $(k = 50, \cos \theta = 1/2) \Rightarrow$

$$L \gtrsim 250, h^* \simeq 50!$$

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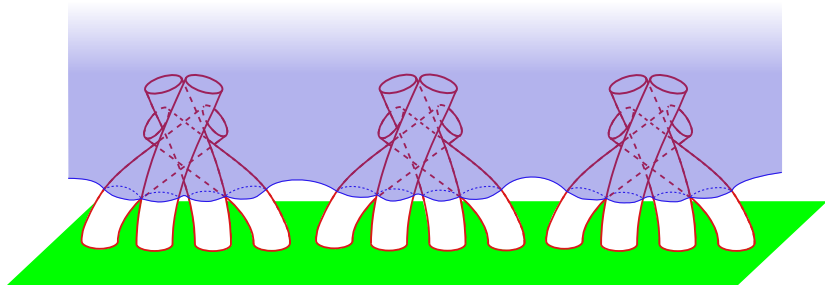
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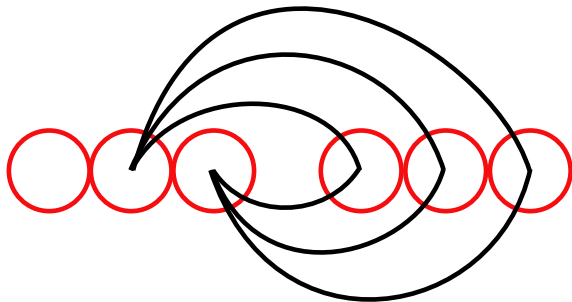
More realistic modeling I

- More than two hairs in a cluster: Even worst conditions.



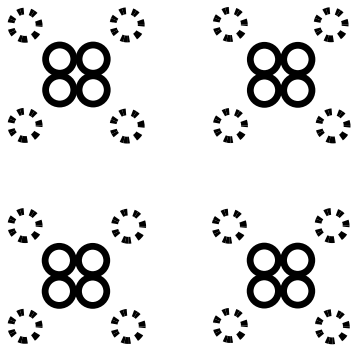
More realistic modeling II

- Pair-wise interactions: Requires hairs with a huge aspect ratio.



More realistic modeling III

- *2D*: Simple model suggests conditions are similar to *1D*.



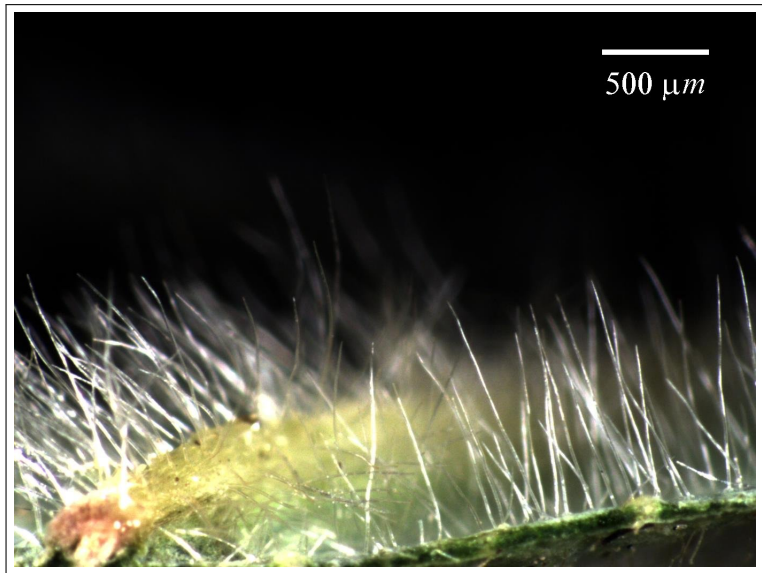
Help from Valentin Blicke



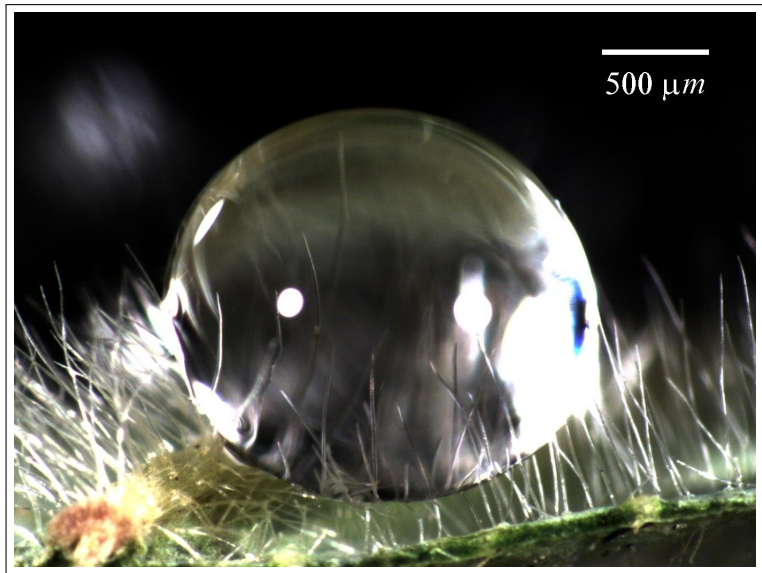
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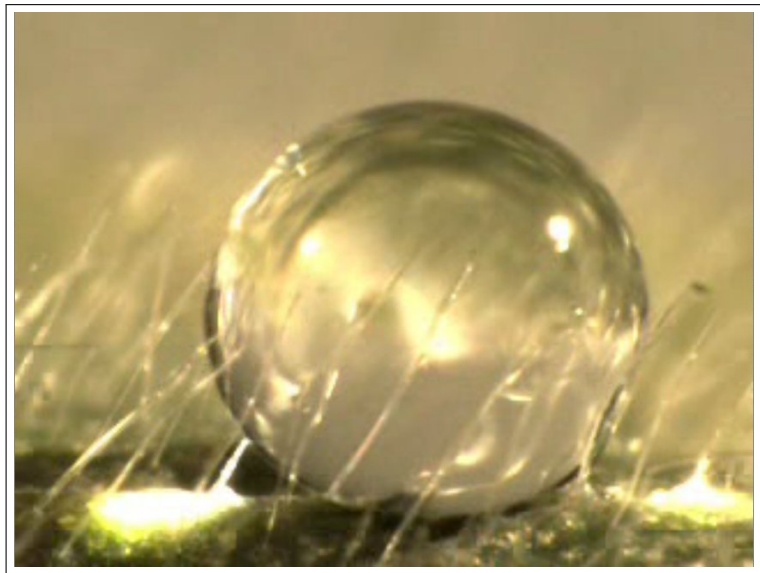
Back to the Lady's Mantle



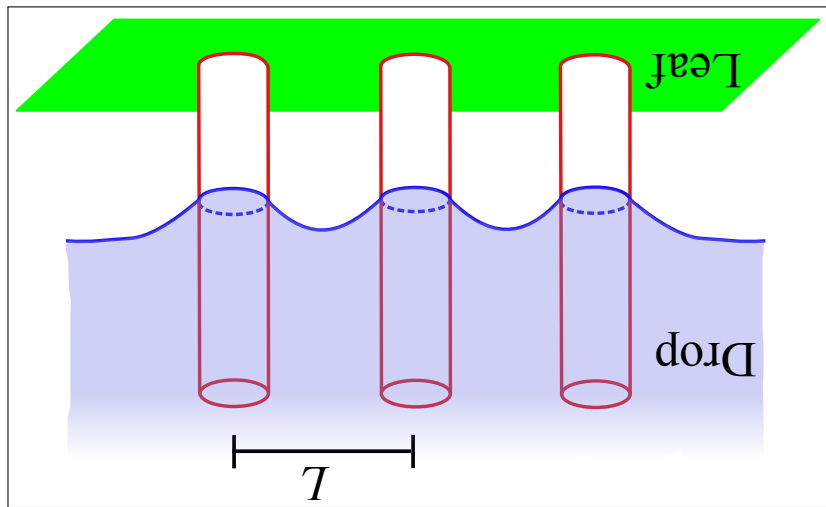
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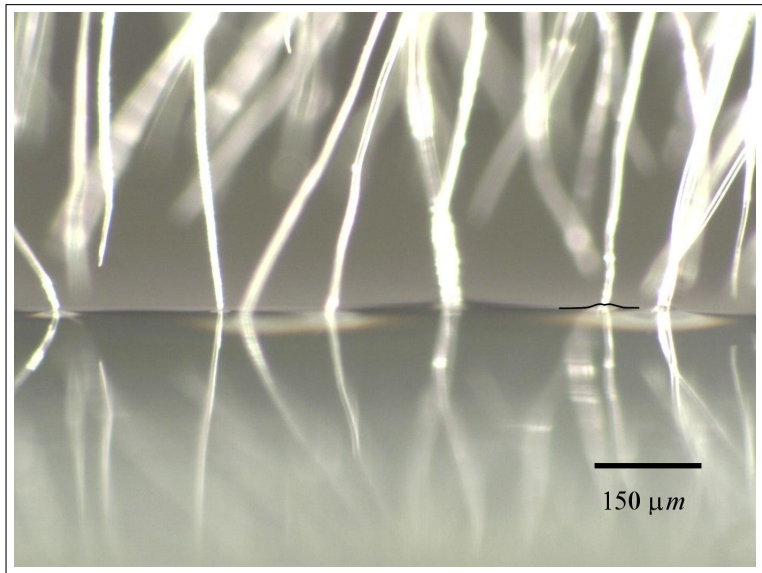
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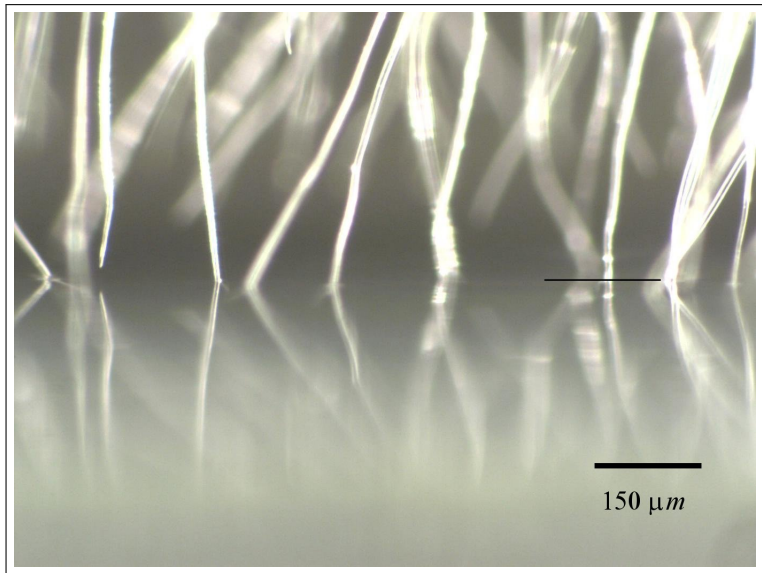
Back to the Lady's Mantle



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Lady's Mantle (*Alchemilla vulgaris*)

“To those who suffer from uncorresponded love, the best is to deposit leaves of this herb on the entrance path to the home of the loved one. But remember, you should deposit them on a New Moon, preferably when the Moon is in Scorpio.”

For those interested

Next New Moon in Scorpio: 6 November 2010, 4:50 GMT.

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