

The Mathematics of Almada Negreiros

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Abstract

In this paper, we deepen our study of the geometric work of Portuguese artist Almada Negreiros, who went beyond the usual practice of geometric abstractionists to find geometric regularities in ways that resemble the work of a mathematician.

Keywords

Euclidean Geometry, Division of the circle, Art, Geometric Abstractionism

Introduction

José de Almada Negreiros (São Tomé and Príncipe, 1893 – Lisbon, 1970) was a key figure of 20th century Portuguese culture, in both visual arts and literature, see figure 1 for a photo. Having spent some time in Paris (1919-1920) and Madrid (1927-1932), where he developed some work with architects, he came back to Portugal, where he married a fellow visual artist, Sarah Affonso. In these years, Almada (as he liked to call himself), having started his artistic life as a cartoonist, he was one of the main artists responsible for introducing the modernist movement in Portugal, being a member of the group *Orpheu*, which included the famous Portuguese author Fernando Pessoa. Identifying himself as Futurist, he engaged in a variety of art forms, challenging the artistic panorama in Portugal: visual art, manifestos, conferences, poetry, plays and ballet. Even though this reference to Futurism waned after the 1920s, he was always an avant-garde figure, always pushing the limits of art forms.

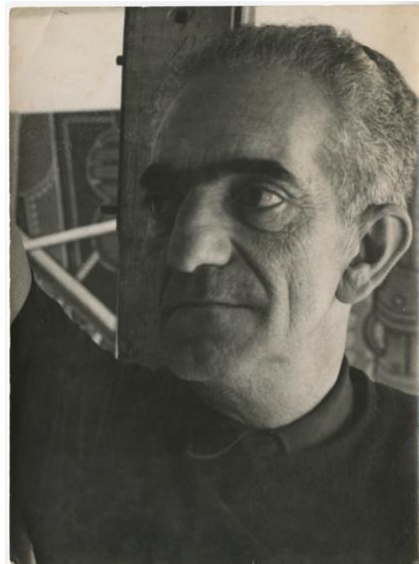


Figure 1. Almada Negreiros in the 1940s

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Upon his return to Portugal in 1932, his work came to be closer to Cubism, and his restlessness was more tempered. He accepted several commissions to work on public buildings, of which the most remarkable are the murals at the maritime stations of Rocha do Conde de Óbidos and Alcântara, the University Campus, in Lisbon, and the Church of Our Lady of Fátima, also in Lisbon (which included stained glass windows). At this time, he continued to develop literary work, namely poetry and a novel, as well as visual works, which started as mostly figurative, such as a portrait of Fernando Pessoa. Only in his later years, from the 1950s, did Almada become interested in geometric abstractionism, as a means to expound his ideas about geometry and art, as we will see — these ideas were presented in two conferences given at BBC radio on this topic, entitled “Theleon and Abstract Art”. With all this work, Almada became one of the leading figures of the art scene in Portugal.

The reason for Almada’s interest in mathematics was mostly philosophical: he wished to show, in his artworks, his personal views on geometry and art. Almada believed there was a set of geometric constructions which would be present in all artworks, in all places and in all times, which he called the *Canon*. We quote from his interviews (Valdemar, 2015).

The canon is not the work of man, it is the possible human capturing of immanence. It is the initial advent of epistemological light.

To move towards a canon. This is the reason for all my work.

[...]

The canon [...] is not just in the examples from the Middle Ages, nor just in the examples from Sumeria, or Crete, Greece, Byzantium, from Arabs or Hebrews, Romanic or Gothic. It exists always, in all places, and this is why it is a canon. And each epoch extracts its rules from the canon.

[...]

All geometric knowledge is of the following kind: the simultaneous division of the circle in equal and proportional parts is the simultaneous origin of the constants of the relation nine/ten, degree, mean and extreme ratio and casting out nines.²

Some of Almada’s geometric work has been known since its creation, in the mid 20th century, such as the *Four abstract paintings* (1957) and the mural *Começar* (1968/9)³, see

² O cânone não é obra do homem, é a captação que o homem pode da imanência. É o advento inicial da luz epistemológica.

Ir encontro a um cânone. Eis a razão de todo o meu trabalho. [...]

O cânone [...] não está só nos exemplos da Idade Média, como não está só nos exemplos da Suméria, não está só nos de Creta, Gregos, Bizantinos, Árabes, Hebraicos, Românicos ou Góticos. Ele está sempre e é por isso mesmo que ele é cânone. E cada época tira do cânone as suas regras. [...]

Todo o conhecimento geométrico é do seguinte teor: a divisão simultanea do quadrado e do circulo em partes iguais e partes proporcionais é a origem simultanea das constantes da relação nove/dez, grau, média e extrema razão e prova dos nove.

³ For a general view of the mural, visit the site <https://gulbenkian.pt/almada-comecar/en/>

figure 2. However, these remarkable works did not shed much light on the author's geometric thinking *per se*, either due to their simplicity (as is the case with the four abstract paintings) or their complexity (the case with the mural *Começar*). We know of this thinking mostly from a written source, the 1960 interviews, conducted by the journalist António Valdemar and published in the Portuguese daily newspaper *Diário de Notícias* (Valdemar, 2015). But a clear materialization of these ideas in the form of artwork was still lacking until recently.



Figure 2. *Começar* by Almada Negreiros. Incised and painted stone, 1968.

Since 2011, the estate of Almada Negreiros has received special attention within the project “Modernismo Online” (<http://modernismo.pt>), and, along time, many unpublished works have resurfaced. Last year, the estate of Almada Negreiros was deposited in NOVA-FCSH, in Lisbon, reorganized and preemptively restored, making it possible to have a global view of this author's work. This is being done within the newly created *Centro de Estudos e Documentação Almada Negreiros e Sarah Affonso* (CEDANSA).

In what concerns geometry (and mathematics in general), hundreds of drawings have been found⁴. We highlight two collections:

- The *Language of the Square* collection, comprising about 128 drawings (some incomplete) on 50 x 70 cm paper. It presents a remarkable formal consistency (in colors and drawing materials) and a content that resembles a book on geometry, progressing from easier to more complicated constructions, with no written explanation of the drawings.
- A collection of 16 small notebooks, measuring 18 x 13 cm, with long folding pages, containing also abstract geometric drawings in progression, along with references to artistic artefacts (mostly paintings) related to these geometric studies.

Figure 3 shows two examples, one from each collection, which will be analyzed in the course of this paper. The original artworks and the images, digitized by António Coelho, belong to the estate of Almada Negreiros. The filing codes for these artworks are ANSA-A-598 and ANSA-C-25-40 — at each point, we will refer to the codes ascribed to each drawing in the catalogue of the estate,⁵ so that the reader can find them at the online archive modernismo.pt.

⁴ See <http://modernismo.pt/index.php/geometria-almada>

⁵ These always start with ANSA, as a reference to “Almada Negreiros and Sarah Affonso”, followed by an A or a C, depending on whether the drawing can be found in an artwork or a notebook, followed by one or more filing numbers.

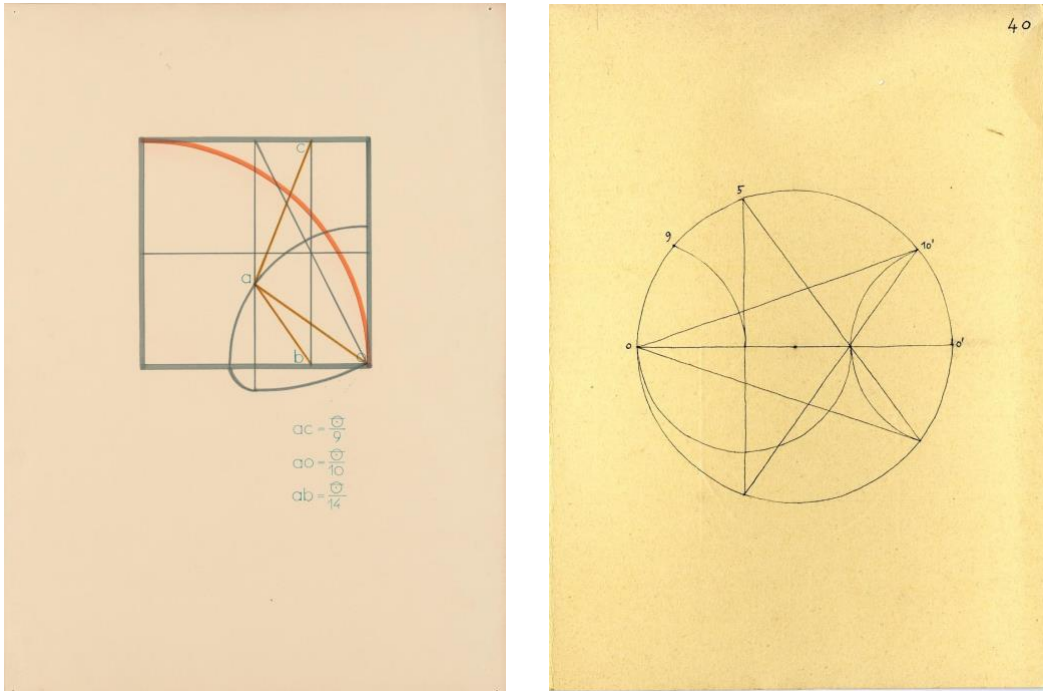


Figure 3. Two artworks by Almada Negreiros, the first from the collection *Language of the Square* (ANSA-A-598) and the second one from a notebook (ANSA-C-25)

These two collections deserve a special reference because of the organization of the geometric material. The sequential order of the drawings, becoming increasingly more intricate, resembles an exposition of a mathematical theory. Other than these, there are also more than a hundred scattered geometric drawings, mostly in squared paper.

We have already started the study of the geometric work of Almada Negreiros, in both articles and books, with Simão Palmeirim (who has a Ph.D. in Fine Arts), see for instance Costa and Freitas (2015b). A systematic approach to these collections and scattered drawings is still lacking, as well as a thorough analysis of the geometric contents of the panel *Começar*. Nevertheless, it is now possible to give a more thorough description of the geometric thinking of Almada Negreiros from his own artistic sources, deepening the results of our previous paper. This is what we propose to do in the following sections.

Constructions and Theorems

We now investigate the geometric drawings of Almada from a mathematical viewpoint. In view of the quotes we have presented previously, along with an analysis of the geometric material that Almada has left us, we can gather that the geometric constructions that constitute what he calls the Canon include the following:

- Divisions of the circle in equal parts,
- Rectangles with definite proportions, such as 3:2, \sqrt{n} (for $n = 2,3,5$) and φ (the golden rectangle),
- The golden section, both in lines and in arcs of circle, and
- The *relation nine/ten*, a particular connection between these two numbers.

Having this in mind, we can see that his geometric drawings constitute attempts to solve two problems:

- To find simple and elegant geometric relations between the elements of the Canon, described above, and
- To apply these canonical elements to the study of artistic artefacts.

In this paper, we will only focus on the first part of his program, the more abstract one. Almada uses mathematical methods, but with an artistic mind, meaning that he presents geometric results, but with no proof other than visual verification. In fact, he expresses this view very clearly in his interviews (Valdemar, 2015):

It is not for scholars that I attempt this publication. I present the result to scholars. I submit myself absolutely to the competence of their respective eruditions. Let them come. But don't bring calculation to a knowledge whose characteristic is not to have it. If the calculations confirm it, congratulations to the calculation. If it does not confirm it, beware the calculation.⁶

This absolute certainty about his body of results can sometimes be found in some scientists, especially when proposing a new promising theory⁷ — but not this contempt for verification, of course.

Nevertheless, unlike what usually happens with geometry-based art, these drawings *do attempt* to prove regularities. Like in Euclid's *Elements*, they can be divided into *constructions* (sequences of ruler-and-compass instructions with an aim to construct a geometric figure or element) and *theorems* (universal regularities, especially concerning the circle). Also, in a mathematical vein, these are presented cumulatively, starting with simpler drawings and elaborating on them, to obtain more sophisticated results. We present examples of both kinds.

The four drawings in figure 4 are reproductions of artworks from the collection “Language of the Square” (the original drawing for the second one appears in figure 1), their codes in the modernismo.pt catalogue are ANSA-A-590, ANSA-A-598, ANSA-A-567 and ANSA-A-898. We preserved the colors and the thickness of the lines, as they help to interpret the content of the drawings. All the drawings in this collection start with a square (hence the name of the collection), with either a circle or a quarter circle inscribed, which is why these elements are drawn with a thicker line.

⁶ Não é para eruditos que tento a divulgação. A eruditos apresento o resultado. Sujeito-me absolutamente à competência das suas respetivas erudições. Que venham. Mas não queiram trazer cálculo a conhecimento cuja característica é não o ter. Se o cálculo confirmar, parabéns ao calculo. Se não confirmar, cuidado com o cálculo.

⁷ One is reminded of a quote attributed to Einstein, about Eddington's measurements at the time of the solar eclipse of May 29, 1919, in case they didn't corroborate his theory: “Then I would have felt sorry for the dear Lord. The theory is correct.”

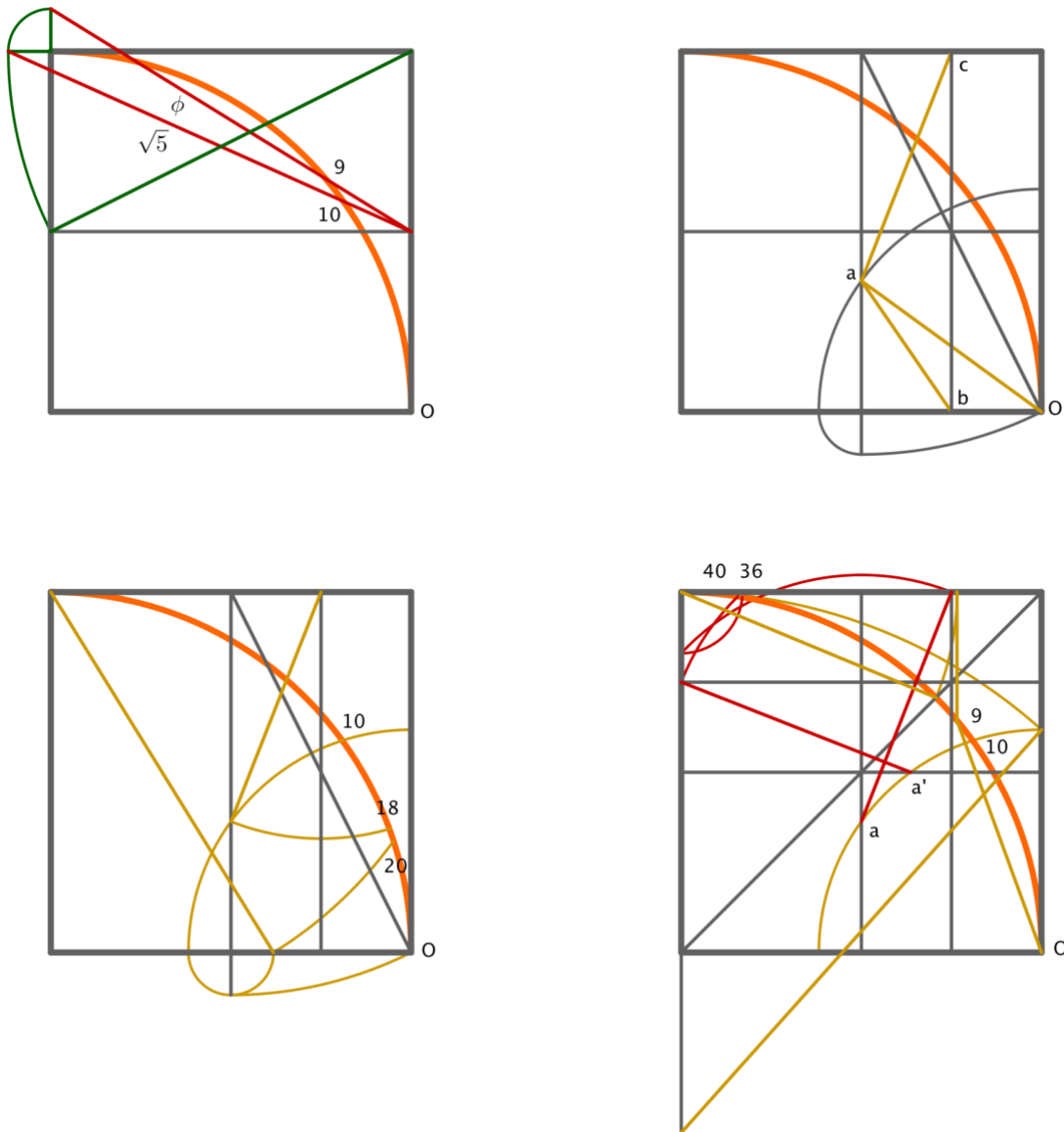


Figure 4. A geometric sequence from *The Language of the Square*

These four drawings, which are part of a sequence of nine, are an example of the development of a geometrical idea, where more elements are added, or new mathematical elements found, all around a point, which the author denotes by a , in the second drawing.

In drawing 1, Almada introduces the geometric construction which will be central for this sequence: from a diagonal of a half-square, he draws two arcs of circle, in green. In this drawing, this leads to two lines in red, marked⁸ ϕ and $\sqrt{5}$. In Almada's notation, this means that if we consider these lines as diagonals of rectangles, with sides parallel to those of the square, these would have the proportions indicated – in particular, one would be a golden rectangle. It's not difficult to verify that these statements are accurate.

⁸ The usual symbol used for the letter phi, when referring to the golden number, is φ , we use this one to follow Almada's choice.

Almada also adds two numbers, 9 and 10, to the intersections of these lines with the quarter circle inscribed in the square. The meaning of these numbers is the following: the arcs from 0 to these points are the 9th and the 10th parts of the circle, respectively. In this drawing, these are not accurate, they are approximations, with an error of about 0.7%.⁹ This is a manifestation of the “relation nine/ten” we have mentioned: the two numbers appear together in a simple and elegant construction.

The notations we find in this drawing are consistent in all of Almada’s geometric work. A number written on a line always represents the proportion of a rectangle, having that line as diagonal, and a number n on a circle always represents the division of this circle in n equal parts.

In the second drawing, we find the two arcs of circle on the lower right. To these, a third one is added, obtaining the point a , marked inside the square, which will be the central point in this series. Almada adds the following annotation to this drawing:

$$ac = \frac{\overline{\odot}}{9} \quad ao = \frac{\overline{\odot}}{10} \quad ab = \frac{\overline{\odot}}{14}$$

This means that the lines ac , ao and ab , all stemming from point a , are the chords of the 9th, the 10th and the 14th parts of the circle, respectively. Only the statement about the 10th part is accurate, even though the one about the 9th part indicates an incredible approximation, with an error of 0.001%, see Freitas (2015) for more details.

The third drawing retains the division of the circle in 10 parts, already obtained, and adds to it the divisions in 18 and 20 parts (which are twice the famous pair 9 and 10), obtained with arcs of circle. The second one is exact, the first one has an error of 0.2%.

Finally, in the fourth drawing, an intricate geometric construction yields the divisions of the circle into 36 and 40 parts — see a detail in the figure 5.

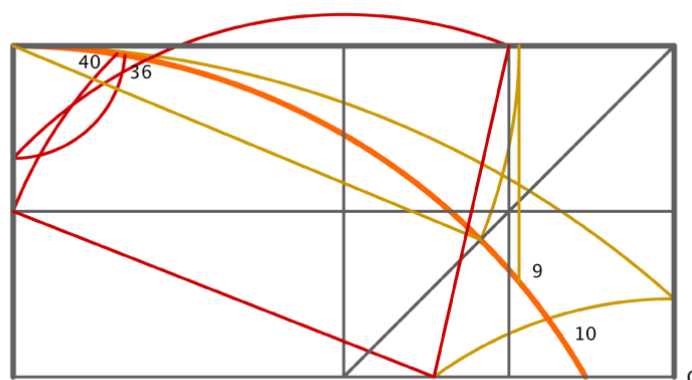


Figure 5. Detail of the last drawing in figure 2

⁹ Many of these drawings have been fully analyzed in Costa and Freitas (2015a), distinguishing the accurate constructions from the approximate ones.

As these are, respectively, 10° and 9° , Almada notes that its difference is one degree, another element of Almada's Canon, as we have seen from the previous quotes. Here, the precision is not so impressive for the 36th part: it has an error of 2.5%. However, for the 40th part, the error is 0.3%.

As we have seen, in this series, Almada presents several divisions of the circle, with each one elaborating on previously obtained results. We recall that this is but a brief example from the collection "Language of the Square", which comprises 128 drawings. A critical edition of this collection is currently being prepared, for which 71 drawings were selected and organized, according to groupings that present this kind of progressive exposition of geometric results.

We now present an example from a notebook, ANSA-C-25 in figure 6, a part of a sequence of five drawings, which includes another element of Almada's canon: the golden section of lines (the original of second image in this figure can be seen in figure 1).

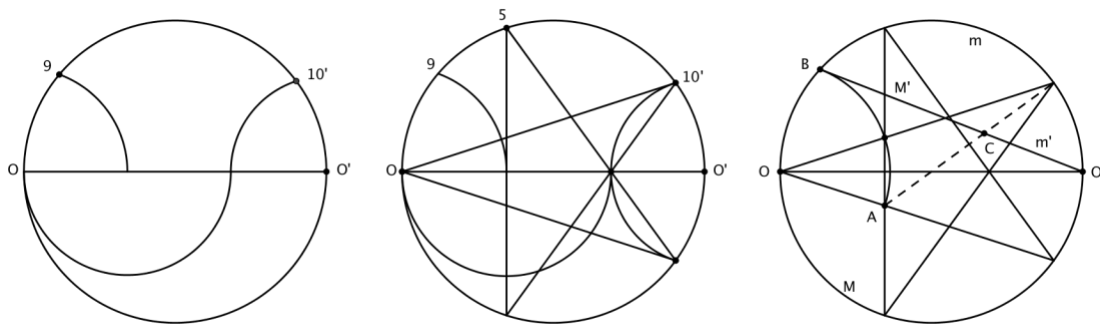


Figure 6. Geometric development from a notebook

The construction suggested in the first image is as follows: given a circle with point 9 already obtained, and a diameter OO' , draw an arc of circle from 9 to the diameter, centered at O , obtaining a new point on the diameter. With center on this point, draw a half-circle, containing point O , one of the extremes of the diameter, determining another point in the diameter. Draw a final arc of circle, with center at the other extreme, O' , passing through this point. This arc will yield the 10th part of the circle.

The accuracy of this construction depends, of course, on the accuracy of the initial point 9. It cannot be fully exact, since, as we will see, the 10th part of the circle can be achieved with straightedge and compass, but not the 9th part. One can check that if one of the markings is exact, the other will have an error of about 1%.

The second image takes advantage of the points already obtained to draw a regular pentagram. Since the exact point 10 is a part of this figure, its accuracy will depend on the accuracy with which this point was determined. It can be proved that if the point 10 is accurate, then the pentagram is regular (Costa & Freitas, 2015a, p. 73).

In the third image, only the pentagram and the circle are kept, and the author proceeds to find two golden sections, in a segment and in the circle. For this construction, one draws an arc of circle, centered at O , passing through A , obtaining point B . Then one draws segments

BO' and the dotted line going through A and C (the point names A, B and C were added to the original drawing). The author claims that the points B and O' determine the golden section of the circle and that point C determined the golden section of line BO', This is signaled by the letters m and M , denoting the small and the large parts of this section (this notation is also consistent throughout his geometric work).

Regarding accuracy, if we start from a regular starred pentagram, and if we take the measure of AB to be 1, the distance between the exact division and the point obtained is 0.0071. As for the golden angle, the accurate value is 137.51° (up to the second decimal), Almada's angle is 137.40° , with an error of 0.08% (Costa & Freitas, 2015a, p. 75).

Almada was aware that, in the pentagram, the intersections of the sides determine the golden section in each other, he mentions the fact in one of his works, the tapestry *Número* (1957). We infer that he's trying to present new occurrences of the ratio in this figure, in this case, both approximated.

As we have remarked, Almada also presents regularities, which we may call "Theorems", bearing in mind that the results are not exact but approximate, in line with his own stated method of work. In the mural *Começar*, he states three of these results, see figure 7.

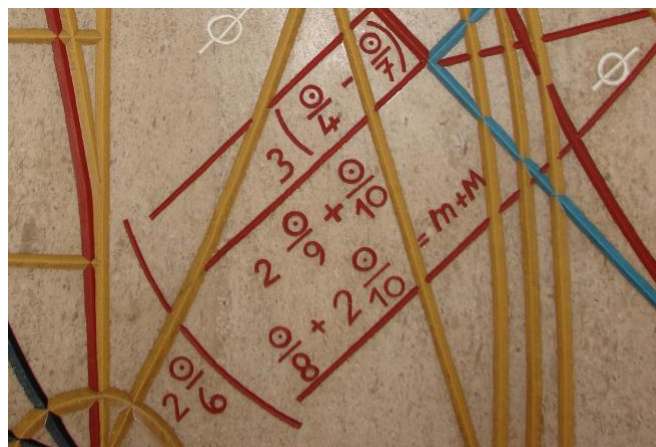


Figure 7. Statements of theorems from the mural *Começar*

The notation $2 \frac{\odot}{6}$ on the lower left refers to the diameter of the circle (two times the chord of the sixth part of the circle, which coincides with the radius). The three expressions on the right are decompositions of this diameter and refer to chords of divisions of the circle in equal parts. We will present Almada's justifications for the first and the second results, we have not yet found one for the third one.

The justification for the first equality appears in a figure in the interviews published in 1960. We reproduce this image in figure 8.

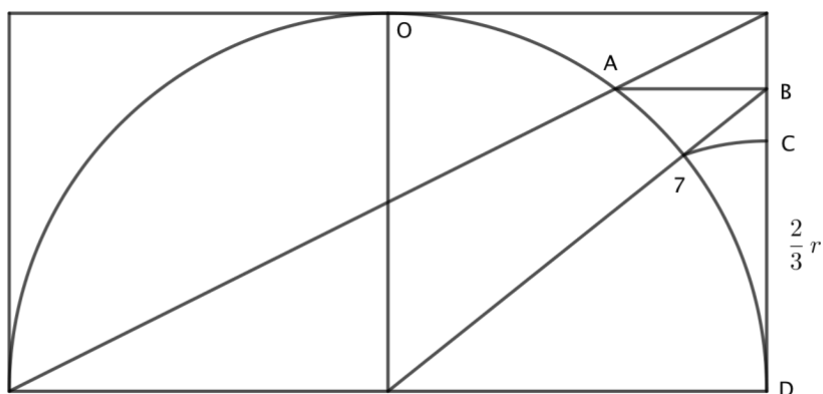


Figure 8. A “theorem” about a decomposition of the diameter of a circle

The only original markings are the points O, 7, and the measure $\frac{2}{3}r$, on the right (all the remaining point names were added for clarity). The construction starts with a double square, with an inscribed semicircle. Drawing the diagonal of the double square, we get point A, and from this point, with a segment parallel to the base of the square, we obtain point B. A line from the midpoint of the base of the rectangle to point B determines point 7, and from this point, a small arc of circle with center D gives us point C.

The arc from O to point 7 is approximately the 7th part of the circle, with an error of 0.2%, and line CD is also a good approximation of two thirds of the radius of the circle, with an error of 0.7%. The statement of the theorem in figure 6 is

$$2 \frac{\odot}{6} = 3 \left(\frac{\odot}{4} - \frac{\odot}{7} \right)$$

This is equivalent to saying that two thirds of the radius is equal to the chord of the arc from point D to point 7, which is what the figure illustrates.

The second result is illustrated by the first image in figure 4:

$$2 \frac{\odot}{6} = 2 \frac{\odot}{9} + \frac{\odot}{10}$$

It states that the diameter of the circle can be decomposed into a chord of the 10th part of the circle, plus two chords of the 9th part. This is again a reference to one of the main elements of the Canon, the “relation nine/ten”. The sum of these three lengths is 0.7% shorter than the diameter.

These are, by no means, the only “theorems” registered by Almada. We have presented these examples of “visual proofs”, so to say, to show Almada’s practice of finding and “proving” geometric regularities, which we can call approximate theorems. We note that this goes far beyond what artists usually do when they use mathematical elements in their visual work: in this case, we have an intent of finding proofs of geometric statements.

We also have no evidence of how Almada would reach these constructions and statements, we can only infer that it was by meticulous drawing and observation, trial and error, done repeatedly over decades. Confirmation would then come by direct measurement, possibly using instruments or exact constructions made on tracing paper.¹⁰

Regular division of the circle

As we have seen, Almada has a very personal style of geometric abstractionism, one that develops geometric results with the aim of revealing what he calls the Canon, a set of geometric constructions that would be primordial to all art. Even though this was primarily an artistic endeavor, his work can also be mathematically appreciated, containing even an advancement to an historical problem: that of dividing the circle in equal parts.

This problem is already treated by Euclid. Book 4 of *The Elements* is dedicated to inscribing several geometric figures in others — namely regular polygons in circles — using straightedge and compass. Euclid shows methods for inscribing the triangle, square, pentagon, hexagon and pentadecagon (15 sides), all regular, in a circle. With angle bisection, one can get more polygons. It is believed that Archimedes also treated this topic, presenting an exact construction for the heptagon, using a ruler with two markings, a style of construction known as *neusis* (Johnson, 1975).

In the Middle Ages, one can find an interest in this topic related to art and artisans. We present two examples. In *A Book on Those Geometric Constructions Which Are Necessary for a Craftsman* (961-972) by Abū'l-Wafā' Al-Būzjānī, Persian mathematician and astronomer (940-998) one finds mathematical studies of this topic, and an approximate construction for the heptagon. The *Sketchbook of Villard de Honnecourt* (medieval architect, 13th century), a very practical image book, includes an approximate construction for the pentagon, applied to the design of the base of a pentagonal tower.

In the Renaissance, Albrecht Dürer (1471-1528), painter, printmaker, and theorist, presented several constructions for the regular division of the circle, some exact, some approximate, in his *Treatise on Mensuration with Compass and Ruler* (Dürer, 1538). The book includes approximate constructions for polygons with 5, 7, 9, 11, 13 and 15 sides.

In 1752, Nicolas Bion, the French king's engineer for mathematical instruments (globes, sundials, mechanical machines, etc.) published the *Traité de la construction et des principaux usages des instruments de mathématique*, a very practical book, which includes a general method for dividing the circle in n equal parts (Bion, 1752). The novelty here is that with just one method, which depended on the division of a line in n parts, one could get a division into any number of parts. The divisions obtained by this method are, in most cases, good approximations.

Gauss was the first to improve on Euclid's list of constructible polygons, by proving, in 1796, that the polygon with 17 sides was constructible. He then proceeded to prove a sufficient

¹⁰ We have seen this method being used today by artists with a similar interest in geometric constructions.

condition for constructability two years later, in the *Disquisitiones Arithmeticae*. Pierre Wantzel proved the necessity of the condition in 1837, achieving the following result.

Gauss-Wantzel's theorem. It is possible to divide a circle into n equal parts, with straightedge and compass, if and only if

$$n = 2^k p_1 \dots p_t$$

where p_1, \dots, p_t are distinct Fermat primes.

A *Fermat prime* is a prime of the form $2^{2^r} + 1$. The only Fermat primes known so far are 3, 5, 17, 257 and 65537.

In particular, we see that it is impossible to divide a circle in 7, 9 or 14 parts: 7 and 14 are divisible by 7, which is not a Fermat prime, and $9 = 3 \times 3$, the Fermat prime 3 appears twice.

Even though this result gave a final answer as to which divisions were constructible, there remained an interest in approximate ones, as some of the methods for the exact ones could be too intricate. In 1853, Housel, a schoolteacher, presented an analysis of the approximations of Bion's method, up to 17 sides (Housel, 1853), with the following remark:

The calculation has been carried out to the seventeen-sided polygon that we now know how to inscribe exactly; but the construction that would result from Mr. Gauss's calculations would be so painful, that this approximation would be even better in practice.¹¹

In the same year, Tempier, another schoolteacher, provided a variation of Bion's method (Tempier, 1853 and Tempier, 1854), which afforded better approximations. For the division of the circle in 17 sides, Bion's method provides an approximation of 2.9%, and Tempier's method, one of 0.2%. For more on this topic, see Freitas and Tavares (2018).

So, as we have seen, throughout history, artisans and non-professional mathematicians have contributed to the problem of the regular division of the circle, accepting both exact and non-exact methods. Almada is thus part of this tradition. His methods provide very reasonable approximations of the regular division of the circle, and in some cases, exact methods—even though these usually are lengthier than the known constructions. However, there is one case in which Almada provided a remarkable new method: the one that provides the division of the circle in 9 parts with an approximation of 0.001%, in drawing 2 of figure 2. This is the best method known to us for the division of the circle in 9 parts (Freitas, 2015), a division that cannot be achieved exactly using straightedge and compass.

Conclusion

¹¹ Le calcul a été poussé jusqu'au polygone de dix-sept cotés que l'on sait maintenant inscrire exactement ; mais la construction qui résulterait des calculs de M. Gauss serait tellement pénible, que cette approximation vaudrait encore mieux dans la pratique.

In the late 19th and early 20th centuries, a speculative use of mathematics took hold in some artistic movements. Even though, in most cases, these developments did not lead to new results in mathematics, they do have a place in the history of the subject, as they reflect a particular way of appropriation of the mathematical concepts, in a non-scientific cultural milieu. Among the most famous 20th century artists who used mathematics explicitly in their production, not as a tool to design their artworks (as is the case with perspective or composition), but as a source of inspiration, or to convey their artistic ideas, are Dalí and Le Corbusier, see Banchoff (2014) and Corbusier (1996). Dalí's *Corpus Hypercubus* (1954), for instance, presents a crucifixion scene with a hypercube, whereas Corbusier's system of measurements *Modulor* is explicitly derived from three mathematical principles: the unit, the double and the golden ratio.

Almada's work in geometric abstractionism places him in this broad movement of encompassing mathematical elements in artworks. His aim is, first and foremost, an artistic one. He never intended to write a geometry handbook, or to propose new mathematical results. His motivations were essentially artistic, although imbued with a philosophical stance: that all art would be based on geometry, on a set of constructions he calls the canon.

This viewpoint led Almada to produce quite a number of geometric works, which, in spite of his own motivations, can be mathematically analyzed. First of all, because his method is similar to that of mathematicians: from simple, clear drawings, Almada builds increasingly more complicated structures, in order to achieve the objectives of his program, namely, to find elegant relations between elements of the canon. Also, besides the method, some of the results presented actually have some mathematical relevance, such as the ones pertaining to the regular division of the circle.

So, we believe, Almada Negreiros holds a very special place, between artists which usually do not provide original mathematical discourses or developments in their work, and amateur mathematicians, which usually do not have artistic motivation when they find new results. A place we believe should be appreciated as particularly unique in the history of mathematics and abstract art.

Acknowledgements

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Bibliography

Banchoff, T. S. (2014). Salvador Dalí and the Fourth Dimension, Proceedings of Bridges 2014: Mathematics, Music, Art, Architecture, Culture.
(<https://www.archive.bridgesmathart.org/2014/bridges2014-1.pdf>)

Bion, N. (1752). *Traité de la Construction et des Principaux usages des Instruments de Mathématique*, 4th ed. Paris: Jombert.

Corbusier, L. (1996). *Le Modulor and Modulor 2*. Birkhauser; 1st edition

Costa, S. P., & Freitas, P. J. (2015a). Livro de Problemas de Almada Negreiros, coleção [Leituras em Matemática](#), vol 14, Portuguese Mathematical Society

Costa, S. P., & Freitas, P. J. (2015b). Almada Negreiros and the geometric canon. *J. Math. Arts* 9: 27–36. (doi.org/10.1080/17513472.2015.1012699)

Dürer, A. (1538). *Underweysung der Messung, mit dem Zirckel und Richtscheyt*. Nüremberg, Germany: Andreae.

Freitas, P. J. (2015). Almada Negreiros and the regular nonagon. *Recreational Mathematics Magazine* 3: 39–50. (<http://rmm.ludus-opuscula.org/Home/ArticleDetails/1125>)

Freitas, P. J., & Tavares, H. (2018). Dividing the Circle, *The College Mathematics Journal*, Volume 49, no. 3, pp. 187-194. (<http://doi.org/10.1080/07468342.2018.1440871>)

Housel, M. (1853). Division pratique de la circonférence en parties égales. *Nouv. Ann. Math.* 12: 77–80. (http://www.numdam.org/item?id=NAM_1853_1_12_345_2)

Johnson, C. (1975). A Construction for a Regular Heptagon, *The Mathematical Gazette*, vol. 59, No. 407 (Mar., 1975), pp. 17-21. (<https://doi.org/10.2307/3616804>)

Tempier, M. (1853). Division pratique de la circonférence en parties égales. *Nouv. Ann. Math.* 12: 345–347. (http://www.numdam.org/item?id=NAM_1853_1_12_345_2)

Tempier, M. (1854). Division pratique de la circonférence en parties égales. *Nouv. Ann. Math.* 13: 295. (http://www.numdam.org/item?id=NAM_1854_1_13_295_1)

Valdemar, A. (2015). Almada. Os painéis, a geometria e tudo – as entrevistas com António Valdemar. Portugal: Assírio e Alvim.